What Fisher Knew About His Relation, We Sometimes Forget[*]

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A B S T R A C T

Expected consumption growth increases the real interest rate as the typical consumer tries to smooth consumption over time. Thus, while consumption growth should be an integral factor in studying the Fisher relation, it is typically excluded from empirical analyses of the relation. This paper demonstrates that *i*) Fisher's view of the role of consumption growth is consistent with a modern understanding of the representative agent's optimization problem via the Euler equation governing the purchase of nominal bonds, and *ii*) the inclusion of consumption growth in the Fisher relation is supported empirically. Finally, our modified Fisher relation provides an alternative method for estimating the consumer's degree of relative risk aversion.

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1. Introduction

Fisher recognized that the *time shape* of income is a determinative factor of the real interest rate.

> "The fact that a person's income is increasing tends to make his preference for present over future income high, as compared with what it would be if his income were flowing uniformly or at a slackening rate; for an increasing income means that the present income is relatively scarce and future income relatively abundant (Fisher, 1930, p. 73-74)."

By 'income' Fisher meant what we now refer to as 'consumption', as he explained in the preliminaries of his book, *The Theory of Interest*, (Fisher, 1930, I.I.61). Most empirical studies of the Fisher relation have generally neglected to include expected consumption growth as an explanatory variable. Although some studies have included income growth, e.g., Levi and Makin (1978), VanderHoff (1984), and Dotsey et al. (2003), the motive for its inclusion was not based upon the idea of income smoothing. In this paper, we derive an augmented Fisher relation from the standard representative agent's Euler equation governing the demand for bonds, which differs from the standard Fisher relation by the inclusion of an expected consumption growth term. We demonstrate that this inclusion is *i*) empirically significant, and *ii*) provides an alternative way to estimate the consumer's degree of relative risk aversion.

The paper is organized as follows: Section 2 elaborates on why and how we should add the consumption growth rate into the Fisher relation, Section 3 empirically examines the implications and interprets the results, and Section 4 concludes.

2. Model

Fisher's quote above is consistent with the analysis of the consumer's Euler equation for bond purchases, which is derived from a standard representative agent problem.

$$\frac{u_c(c)}{p} = \beta\left(1+\tilde{i}\right)E\left[\frac{u_c(c')}{p'}\right] \tag{1}$$

Here a prime denotes variables measured at time $t + 1$, a tilde distinguishes the interest rate in levels from the log of one plus the interest rate used in some of the equations below and E is the expectations operator at time t. The optimizing consumer equates the marginal utility value of current consumption with the discounted expected future utility value consumption in the next period. Assuming a constant degree of relative risk aversion utility function,

$$u(c) = \begin{cases} \dfrac{1}{1-\gamma} c^{1-\gamma} & \gamma \neq 1 \\[2mm] \ln c & \gamma = 1 \end{cases} \tag{2}$$

Equation (1) becomes

$$\left(1+\tilde{i}\right)^{-1} = \beta\, E_t \left[\frac{p}{p'} \cdot \left(\frac{c}{c'} \right)^{\gamma} \right] \tag{3}$$

where γ represents the consumer's degree of relative risk aversion. Expanding the expectation term we obtain

$$\left(1+\tilde{i}\right)^{-1} = \beta \left\{ E_t\left(\frac{p}{p'} \right) E_t\left(\frac{c}{c'} \right)^{\gamma} + \mathrm{cov}\left[\frac{p}{p'}, \left(\frac{c}{c'} \right)^{\gamma} \right] \right\} \tag{4}$$

To simplify this expression we define the following terms

$$\rho = \ln \beta^{-1}, \; i = \ln\left(1+\tilde{i}\right), \; \pi^e = -\ln E\left(\frac{p}{p'} \right), \; g^e = -\ln E\left(\frac{c}{c'} \right) \tag{5}$$

where ρ represents the consumer's rate of time preference, i represents the nominal interest rate, π^e is the expected inflation rate, and g^e represents the expected consumption growth rate.[1]

Assuming that the covariance term in Equation (4) is negligible, as expected in a low inflation risk economy (Sarte, 1998), we obtain our augmented Fisher equation by taking the natural log of both sides of Equation (4) and substituting in the terms from Equation (5):

$$i = \rho + \pi^e + \gamma g^e \tag{6}$$

The nominal interest rate must compensate the consumer for *i*) the rate of time preference, *ii*) the loss in purchasing power due to inflation, and *iii*) the utility cost of expected consumption fluctuations. Note that the coefficient on the consumption growth rate is the consumer's degree of relative risk aversion. An estimation of Equation (6) provides us with an alternative way to measure this parameter.

3. Data and Estimation

In testing the implications of our model, we use US quarterly data for 3-month Treasury constant-maturity bonds (Board of Governors), seasonally-adjusted real personal consumption expenditures (Bureau of Economic Analysis), and the consumer price index for urban

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[1] The proper inflation measure is the 'inverse of the expected rate of decrease in purchasing power due to inflation' rather than the 'rate of inflation' itself. According to Jensen's inequality these are not the same in the presence of risk. The same also applies to the consumption growth rate. In this study we proxy for the measure of inflation and consumption expectations by a single point, namely the actual inflation or growth, so there is no difference between $-\ln E\left(p/p' \right)$ and $\ln E\left(p'/p \right)$ or $-\ln E\left(c/c' \right)$ and $\ln E\left(c'/c \right)$.

consumers (Department of Labor) for the sample period of 1960Q4 to 2005Q4. Inflation and consumption growth expectations are proxied for by the actual 3-month growth rates of CPI and consumption data, respectively. Instrumental variable estimation is used to overcome potential endogeneity problems caused by the proxies. Also, to avoid artificial moving average problems, we select non-overlapping data points for (3-month) inflation and consumption growth and also divide the T-Bill rates by four (Sun and Phillips, 2004). Finally, since the Treasury rate reflects the yield to be collected one quarter after purchase, we align next quarter's inflation and consumption growth rates with the current quarter's interest rate (Sun and Phillips, 2004).

The standard cointegration techniques by Crowder and Hoffman (1996), Ng and Perron (1997), Dotsey et al. (2003), Sun and Phillips (2004), and Caporale and Pittis (2004) do not perfectly fit the purposes of our study since the consumption growth component is found to be stationary,[2] and is used to explain the short run fluctuations around the long-run Fisher relation. We follow Mehra (1993) who uses an error correction method to tackle a similar problem for the joint estimation of the long- and short-run money demand function. While our long-run Fisher equation is

$$i_t = \rho_0 + \rho_1 \pi_t + \rho_2 g_t + u_t \tag{7}$$

its short run adjustment process incorporates the dynamic error correction changes:

$$\Delta i_t = \theta_0 + \sum_{s=0}^{n1} \theta_{1,s} \Delta \pi_{t-s} + \sum_{s=0}^{n2} \theta_{2,s} \Delta g_{t-s} + \theta_3 u_{t-1} + \varepsilon_t \quad . \tag{8}$$

where u_t and ε_t are deviations from long and short run equlibriums, respectively. Estimating both equations simultaneously in a reduced form equation yields:

$$\Delta i_t = \psi_0 + \sum_{s=0}^{n1} \psi_{1,s} \Delta \pi_{t-s} + \sum_{s=0}^{n2} \psi_{2,s} \Delta g_{t-s} + \sum_{s=0}^{n3} \psi_{3,s} \Delta i_{t-s} - \psi_4 \pi_{t-1} - \psi_5 g_{t-1} + \psi_6 i_{t-1} + \varepsilon_t \tag{9}$$

which provides us with consistent estimates of the structural parameters of interest $\theta_3 = \psi_6$, $\rho_0 \cong -\psi_0 / \psi_6$ (since $\psi_0 = \theta_0 - \theta_3 \rho_0$), $\rho_1 = -\psi_4 / \psi_6$ and $\rho_2 = -\psi_5 / \psi_6$. Due to possible endogeneity resulting from proxying for expectations, we estimate the model with both OLS and IV methodologies.[3] The results are displayed in Table 1.[4]

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[2] ADF tests, using the modified Akaike information criteria for lag selection, find that the T-Bill and inflation rates have a unit root, while rejecting the non-stationarity of consumption growth; the corresponding test statistics are -1.87, -2.38, and -5.92, respectively.
[3] In OLS, n1, n2 and n3 are all chosen as 1 while they are 2 in the IV estimation. They are determined by using the Schwartz information criterion. The instruments used in the IV model are 2 non-coincident lagged levels of the T-Bill rate, inflation and consumption growth with 2 coincident lagged differences of the interest rate and 4 lagged differences of inflation and consumption growth.

(Insert Table 1 here)

      The first two columns of Table 1 show the reduced form estimates while the latter two display the structural parameters of Equation (7) derived from the reduced form estimates. While we use standard t-statistics to calculate the significance levels of reduced form coefficients, we use a combination of Wald and t-statistics to test for null hypotheses of the long-run Fisher relation, namely $\rho_1 = 1$ ($\psi_6 + \psi_4 = 0$) and $\rho_0, \rho_2 = 0$ ($\psi_5 = 0, \psi_0 = 0$). The findings support our theory that consumption growth is an important element in the Fisher relation explaining the short-run fluctuations around the long-run Fisher relation. While the long-run inflation coefficient is not significantly different than one and the time preference rate is insignificantly different than zero, consumption growth is marginally significant. Its coefficient, the degree of relative risk aversion is within the acceptable range of values in the literature.[5] This last finding is not surprising considering the expected value of consumption growth is a constant part of the real rate in the limit. In addition to the marginal significance in the long-run Fisher relation, the dynamic error correction estimates show that consumption growth changes are important in explaining the adjustments in the interest rate as well as their level.

4. Conclusion

The effect of expected consumption growth on the interest rate has been overlooked in studies of the Fisher relation. We find that including a consumption growth term in the estimation of the Fisher equation yields a significant coefficient for consumption growth and allows us to estimate the consumer's degree of relative risk aversion; therefore, we conclude that the future studies of the Fisher relation would benefit from including expected consumption as an explanatory variable.

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[4] Since the estimates of the standard Fisher relation in the literature are not too different than our estimates, we refrain from including them in Table 1 in the interest of brevity.

[5] The range of CRRA estimates range from 0.5 to 2.5.

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Table 1: Reduced Form and Structural Parameters from the Joint Test of the Long- and Short-Run Fisher Relation

	OLS (reduced)	IV (reduced)	OLS (structural)	IV (structural)
ψ_0	-0.04 (0.06)	-0.08 (0.06)	-0.37	-0.82
π_{t-1}	0.14^{***} (0.03)	0.14^{***} (0.03)	$1.23^{\dagger}$	$1.46^{\dagger}$
g_{t-1}	0.06^{*} (0.03)	0.07^{*} (0.04)	0.51^{*}	0.77^{*}
i_{t-1}	-0.11^{***} (0.03)	-0.09^{***} (0.03)		
$\Delta\pi_t$	0.10 (0.03)	0.11^{***} (0.04)		
$\Delta\pi_{t-1}$	0.07^{***} (0.03)	0.07 (0.04)		
$\Delta\pi_{t-2}$		0.004 (0.03)		
Δg_t	-0.01 (0.02)	-0.01 (0.02)		
Δg_{t-1}	-0.06^{***} (0.02)	-0.09^{***} (0.03)		
Δg_{t-2}		-0.02 (0.01)		
Δi_{t-1}	-0.03 (0.07)	-0.12 (0.08)		
Δi_{t-2}		-0.09 (0.09)		
$\bar{R}^2$	0.27	0.26		
Prob. of J stat		0.99		
DW	2.11	1.96		

Notes: Standard errors are reported in the parentheses. *** indicates 99% significance while $^{**}(^{*})$ indicates 95%(90%). The structural parameters are estimated using the reduced form estimates, and their significance levels are tested using Wald ($\psi_6 + \psi_4 = 0$) and t-tests ($\psi_5 = 0, \psi_0 = 0$). † indicates failure to reject the null of the coefficient equaling 1.