# The (Ir)relevance of Inflation Persistence for Inflation Targeting Policy Design

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**Abstract**

This paper analyzes the relevance of sectoral inflation persistence differentials for optimal monetary policy using a two-sector sticky price model, which generalizes the models existing in the literature by introducing inflation persistence to both sectors. The results show that even if the sectors have the same degree of inflation persistence, optimal inflation targeting policy attaches different weights to these. In particular, different combinations of price change frequency and backward looking price setting parameters can produce the same inflation persistence but have different implications for the optimal inflation targeting policy. It is also shown that targeting the inflation of the more persistent sector is not robust to different calibrations of parameters of the price setting mechanism. However, optimal inflation targeting rule attaches a higher weight to the inflation of the sector with a flatter Philips curve whether it is more persistent or not. Having derived a central bank loss function as a second order Taylor approximation to the welfare, I show that the optimal inflation targeting rule fails to approximate the optimal policy when sectoral inflations are persistent and the welfare loss increases as the inflation persistence differential across sectors increases.

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# 1 Introduction

The existence of heterogeneity in inflation persistence across sectors is well documented.[1] This evidence has important implications for monetary policy. Understanding sectoral responses to monetary policy shocks can be helpful in explaining the mechanism through which monetary policy affects the real economy.[2] Moreover, the heterogeneity across sectors determines the way monetary policy should be designed since the hands of the policy makers are tied with a single policy instrument to control for the developments in different sectors. Under an inflation targeting regime the particular question is how the existence of different degrees of inflation inertia across sectors affects the design of the target inflation measure. That is, which measure of inflation should the central bank target? This paper answers that question in a very general two sector framework.

Sectoral differences in firms' price setting mechanisms is generally thought of as the source of the heterogeneity in inflation persistence. The relevance of heterogeneity in price setting mechanism is analyzed before by assuming different price change frequencies for different sectors. Aoki (2001) employs a model with one flexible and one sticky price sector and shows that optimal policy is to stabilize a core inflation measure given by the inflation of the sticky price sector. Benigno (2004) introduces sluggish price adjustment à la Calvo into both sectors and argues that optimal inflation targeting policy is to attach higher weight to the inflation of the sector that is constrained by a lower frequency of price change.[3] Lower frequency of price change also implies a flatter New Keynesian Philips curve (NKPC). Thus, the higher is the ability of the sectoral inflation to adjust to the efficient fluctuations in the output gap, the smaller is the weight attached to its inflation.

Note that these models imply purely forward looking NKPC and produce front-loaded

[1] For univariate analysis see, among many others, Aucramanne and Collin (2005), Altissimo, Mojon and Zaffaroni (2007), Bilke (2004), Lünnemann and Mathä (2004).

[2] See Carvalho (2005) and Barsky et al. (2007).

[3] Note that the analysis of the optimal monetary policy under a currency union with heterogeneous regions and in a single country with heterogeneous sectors is analogous. The only difference is that what is called terms of trade in the two region model corresponds to a relative price in the two sector model.

impulse responses. Thus, as suggested by Mankiw and Reis (2002) and Fuhrer and Moore (1995) for single sector models, they do not capture the persistence of inflation observed in the data. An exception to this is Benigno and Lopez-Salido (2006) who develop a two-sector sticky-price model with a single sector displaying inflation persistence. The persistence in that sector is modeled by introducing a type of producers who set their prices according to a rule-of-thumb consistent with a similar single sector model of Gali and Gertler (1999).[4] Benigno and Lopez-Salido (2006) concludes that it is optimal to target the inflation of the sector that is persistent. Noting that the persistence in the purely forward looking sector is zero, Levin and Moessner (2005) argue that result of Benigno and Lopez-Salido (2006) can be interpreted as targeting the inflation of the sector that has *higher* degree of inflation persistence. Therefore, the main concern of this paper is that whether targeting the inflation of the more persistent sector is a robust policy implication in a generalized model.

In this paper, I extend the analysis to take account of sectoral differences in inflation persistence by assuming backward looking price setters for both sectors. That is, different than the ones implied by standard models, the NKPC of each sector links the current inflation to that of the previous period as well as to expected inflation and the output gap.[5] Although evidence suggests that all sectors are characterized by hybrid price setting with different mechanisms, currently there exists no study allowing for inflation persistence for both sectors and conducting analysis for different feasible values of the price setting parameters. Thus, one of the goals of this paper is to fill this hole in the literature.

First, the implications of the equal degree of inflation persistence across sectors is analyzed. An equal degree of inflation persistence can be produced with different combinations of

[4] In an earlier version Benigno and Lopez-Salido (2002) propose an approximate nominal rigidity measure for the hybrid price setting sector and suggest that when this measure implies the same degree of nominal rigidity across sectors, optimal inflation targeting policy is targeting the CPI inflation. For feasible calibrations, Kösem-Alp (2009) shows that targeting CPI inflation on the basis of equivalence of this measure across sectors implies significant welfare loss.

[5] Leith and Malley (2005) and Massidda (2005) shows that the coefficient of the lagged inflation in the NKPC is significant and different across sectors and that this heterogeneity arises from different price change frequencies and different fractions of backward indexing producers.

frequency of price change and fraction of backward looking price setters. This analysis is done in order to understand whether homogeneity in degree of inflation persistence can be a summary statistic and help the central bank avoid considering the underlying sectoral heterogeneities. If so, equal persistence across sectoral inflations would imply the optimality of the CPI targeting. Results, however, show that CPI targeting is optimal only if the sectors have exactly the same price setting mechanisms.

Next, the relevance of the higher persistence in one sector for optimal inflation targeting is studied by calibrating one of the sectors to be more persistent than the other. These results show that targeting the inflation of the sector that is more persistent is not a parameter robust policy and for some parameter combinations it is optimal to target the inflation of the less persistent sector. Moreover, I find that optimal weights are determined according to the relative slope of the NKPC of the sectors, rather than the relative persistence across sectors. That is, as the inflation of a sector becomes less elastic to the changes in the output gap, the weight attached to the inflation of that sector increases whether its inflation is more persistent than that of the other sector or not. Thus, the result of Benigno (2004) carries over to the models with backward looking price setters and it is the slope of the NKPC rather than the inflation persistence, that matters for the optimal inflation targeting policy.

As far as welfare implications of inflation targeting are concerned, as shown by Rotemberg and Woodford (1998), in single sector models without inflation persistence inflation targeting policy coincides with the optimal policy. Inflation targeting policy still approximates the optimal policy in a two sector environment unless the inflations of the sectors are persistent.[6] However, when inflation persistence is introduced the optimal inflation targeting policy fails to approximate the optimal policy. For some parameter combinations, optimal inflation targeting rule implies a welfare loss almost twice as high as that of optimal commitment rule and exhibits significantly different impulse responses. Here

[6] See Eusepi et al.(2009)for a multi sector analysis.

the welfare loss under each policy is computed according to a central bank loss function, which is derived as a second order approximation to the social welfare. Moreover, a higher level of inflation persistence in the economy as a whole implies a higher or lower additional welfare loss depending on its source. I also find that the additional welfare loss implied by optimal inflation targeting increases as the magnitude of the difference between the persistence of sectoral inflations increases. That is, the welfare loss is smallest when the sectors have same degree of inflation persistence.

The rest of the paper is structured as follows. In Section 2, I describe the measure of the inflation persistence used throughout the paper. Section 3 presents the model and the utility based welfare function that policymakers seek to maximize. The emphasis will be on how the existence of backward looking price-setters affects this welfare function. Section 4 shows the optimal inflation targeting rule under homogenous and heterogeneous degrees of inflation persistence across sectors. Section 5 displays the welfare implications of adopting optimal inflation targeting instead of the optimal commitment policy and impulse response analysis. Section 6 concludes.

# 2 Measuring the Inflation Persistence

Aggregate or disaggregated inflation persistence is usually computed under univariate models by summing the autoregressive coefficients. Under this reduced form analysis the sources behind the inflation persistence are not clear. Thus for disaggregated series, the source of heterogeneity is not obvious. Angeloni et al. (2004) address this issue and distinguish the possible sources of inflation persistence by employing a hybrid NKPC of the following form:

$$\pi_t = \kappa_1(y_t - y_t^n) + \kappa_2\pi_{t-1} + \kappa_3\pi_{t+1} + u_t$$

where $\pi_t$ is inflation and $(y_t - y_t^n)$ is output gap. Intrinsic persistence is given by $\kappa_2$, which measures the dependence on past inflation due to the price setting mechanism.[7]

[7] Levin and Moessner (2005) show that as the degree of endogenous persistence increases, the reduced form measure of inflation, the serial correlation in inflation also increases.

Since the focus of this paper is the heterogeneity in inflation persistence arising from the existence of different price setting mechanisms across sectors, the measure of inflation persistence used in this paper is $\kappa_2$.[8] Following Levin and Moessner (2005), I compute the serial correlations in sectoral inflations for different degrees of endogenous persistence in sectoral inflations both under the optimal commitment policy and the simple Taylor rule of the form:

$$i_t = 1.5\pi_t + 0.5(Y_t - Y_t^n) + e_t$$

Panels of Figure 1 display the difference in serial correlations in inflation persistence across sectors as a function of the difference in degree of endogenous persistence. The figure shows that the sector with a higher degree of endogenous persistence also has a higher degree of reduced from inflation persistence and as the difference in endogenous persistence increases, the difference in serial correlation in sectoral inflations increases. Moreover, this finding is robust not only to the choice of the monetary policy rule but also to alternative calibrations of the sectoral price setting mechanisms, which will be explained in detail in the following section. Therefore, within the set up of this paper, the word intrinsic is redundant and I refer to the sector with a higher $\kappa_2$ as more persistent.

# 3 The Model

The model studied in this paper is a standard stochastic general equilibrium representative household model with two monopolistically competitive sectors. The sectors are assumed to be of the same economic size.[9] Both sectors are characterized by sluggish price adjustment and a fraction of producers in each sector are unsophisticated price setters, who adjust their prices to according to a rule of thumb. In this paper, I generalize the standard two sector models in the literature by introducing backward indexing producers

[8] In order to differentiate the intrinsic inflation persistence from extrinsic persistence, the sources of inflation persistence exogenous to the price setting, I calibrate the sectors with equally persistent supply shocks.

[9] Alternative calibrations of the share in consumption or equivalently the economic size is also possible and do not change the results.

into both sectors.

### 3.1 Utility of a Representative Household

Each household consumes all of the differentiated goods in both sectors, and produces a single good. The objective of household $j$ is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(\xi_t^d C_t^j) - v(\xi_{i,t}^s y_{i,t}^j) \right] \tag{1}$$

where $u(\cdot)$ represents the utility of consumption and $v(\cdot)$ represents the disutility of production. I make the usual assumptions that $u(\cdot)$ is increasing and concave, and that $v(\cdot)$ is increasing and convex. The constant $\beta \in (0,1)$ is the discount factor and the argument $C_t^j$, which represents a CES index of representative household purchases of the differentiated goods of both sectors, is defined as

$$C_t^j = \frac{1}{2} \left( C_{1,t}^j \right)^{1/2} \left( C_{2,t}^j \right)^{1/2} \tag{2}$$

where $C_{i,t}^j$ itself is a CES aggregate of sectoral goods.

$$C_{i,t}^j = \left[ \int_0^1 c_{i,t}^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

Here $i \in \{1, 2\}$ indexes sectors. The elasticity of substitution between any two differentiated goods in each sector, $\theta$, is assumed to be greater than unity and uniform across sectors. The argument $y_{i,t}^j$ is the output of the good that representative household $j$ in sector $i$ produces. The household indexed by $j$ produces one of the differentiated goods in sector $i$. Following Aoki (2001), I assume that the preference shock $\xi_t^d$ is identical across all households. I also assume that $\xi_{i,t}^s = \xi_{1,t}^s$ for all households producing one of the differentiated goods of the first sector and $\xi_{i,t}^s = \xi_{2,t}^s$ for all households producing one of the differentiated goods of the second sector, where $\xi_t^d$ and $\xi_{i,t}^s$ are stationary random shocks. These assumptions imply that all of the producers in each sector face the same supply shocks and that there is no sector specific demand shock in this economy.

### 3.2 The Consumption Decision

The model assumes complete financial markets with no obstacles to borrowing against future income, so that each household faces a single intertemporal budget constraint. The model further assumes that households can insure one another against idiosyncratic income risk. These assumptions imply that, if all households have identical initial wealth, they will choose identical consumption plans. The optimal allocation for a given level of nominal spending across all of the differentiated goods of both sectors at time $t$ leads to the Dixit-Stiglitz demand relations as functions of relative prices. For the following, the index $j$ is suppressed, since the consumption decision is identical across all households. The total expenditure required to obtain a given level of consumption index $C_t$ is given by $P_tC_t$, where $P_t$ is defined as

$$P_t = (P_{1,t})^{1/2}(P_{2,t})^{1/2} \tag{4}$$

Here $P_{i,t}$ is the price index of the sector $i$ defined below. I assume that the share of each sectors' composite good comprises the half of total consumption. Demand for the sectoral composite differentiated goods of sector $i$ are the usual Dixit-Stiglitz demand relations as functions of relative prices, which are given by

$$C_{i,t} = \frac{1}{2}\left(\frac{P_{i,t}}{P_t}\right)^{-1} C_t \tag{5}$$

where $P_{i,t}$ is the Dixit-Stiglitz price index defined as

$$P_{i,t} = \left[\int_0^1 p_{i,t}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \tag{6}$$

where $p_{i,t}(z)$ is the price of differentiated good in sector $i$ indexed as $z$ at time $t$. Demand for each differentiated good z, $c_{i,t}(z)$, is given by

$$c_{i,t}(z) = \frac{1}{2}\left(\frac{P_{i,t}}{P_t}\right)^{-1}\left(\frac{p_{i,t}(z)}{P_{i,t}}\right)^{-\theta} C_t \tag{7}$$

The optimal consumption plan of the household must satisfy

$$\frac{\xi_t^d u'(\xi_t^d C_t)}{P_t} = \Lambda_t \tag{8}$$

where Λt is marginal utility of nominal income, which follows the rule of motion

$$\Lambda_t(1 + \mathrm{R_t}) = \beta\Lambda_{t+1} \tag{9}$$

with $R_t$ the risk-free nominal interest rate at time $t$.

## 3.3 The Production Decision

It is assumed, as is standard in this literature, that prices in both sticky-price sectors are changed at exogenous random intervals in the fashion of Calvo (1983). The producers in each sector can change their prices with a constant probability $1 - \alpha_i$. A fraction $1 - \psi_i$ of the households who can change their prices behave optimally when making their pricing decisions. I refer to these producers as the forward-looking households. The remaining producers, a fraction $\psi_i$, instead use a simple backward-looking rule-of-thumb when setting their prices. I refer to these producers as the backward-looking producers.

Given the complete markets and symmetric initial steady state assumptions, all forward-looking producers that are able to adjust their price at date $t$, will choose the same price. Let $P_{i,t}^f$ denote this price. I assume that all backward-looking producers who change their price at date $t$ also set the same price. Let $P_{i,t}^b$ denote this price.

The forward looking producer who is able to choose his price in period $t$ chooses $P_{i,t}^f$ to maximize the discounted future profits

$$E_t\left\{\sum_{k=0}^{\infty}\left\{(\alpha_i\beta)^k[\Lambda_{t+k}p_{i,t}y_{i,t+k} - v(\xi_{t+k}^s y_{i,t+k}\right\}\right\} \tag{10}$$

First term is the expected revenue in utility terms. Since the cost of production is in terms of utility, the revenue is multiplied by the marginal utility of income. Maximizing the objective function with respect to $P^f_{i,t}$ gives the following first order condition:

$$E_t \left\{ \sum_{k=0}^{\infty} \left\{ (\alpha_i \beta)^k \Omega_{i,t+k} (P^f_{i,t} - \frac{\theta}{\theta - 1} S_{i,t+k} \right\} \right\} = 0 \tag{11}$$

where

$$\Omega_{i,t+k} \equiv \frac{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})}{\xi^d_t u'(\xi^d_t C_t)} c_{i,t+k}(z) \tag{12}$$

and

$$S_{i,t+k} = \frac{\xi^s_{i,t+k} v'(\xi^s_{i,t+k} y_{i,t+k})}{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})} P_{i,t+k} \tag{13}$$

is interpreted as the nominal marginal cost of sector $i$. Since the household is both worker and the owner of the firm in sector $i$, the cost of production is the disutility resulting from working.

As in Gali and Gertler (1999), I assume that the backward-looking firms set their prices according to the following rule:

$$P^b_{i,t} = P^*_{i,t-1} \pi_{i,t-1} \tag{14}$$

where $\pi_{i,t-1}$=$p_{i,t-1}/p_{i,t-2}$ and $P^*_{i,t-1}$is an index of prices set at *t-1,*given by

$$P^*_{i,t-1} = (P^f_{i,t-1})^{1-\psi_i} (P^b_{i,t-1})^{\psi_i} \tag{15}$$

According to equation (14) the backward looking firms adjust their prices to equal the geometric mean of the prices that they saw chosen in the previous period, $P^*_{i,t-1}$, adjusted for the sectoral inflation rate they observed in the previous period, $\pi_{i,t-1}$. That is, these firms use the inflation observed in the previous period a proxy for that of the current period. This way of price setting, while not optimal, keeps their relative prices same across periods when inflation is constant, for example at steady state.

The aggregate price level will then evolve according to

$$P_{i,t} = \left(\alpha_i P_{i,t-1}^{1-\theta} + (1-\alpha_i)(1-\psi_i)(P_{i,t}^f)^{1-\theta} + (1-\alpha_i)\psi_i(P_{i,t}^b)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{16}$$

Each period, a fraction $\alpha_i$ of the producers keeps charging the price of the previous period. The remaining $1-\alpha_i$ of the firms change their prices but only $1-\psi_i$ of them choose the optimal price and the remaining producers set their prices according to the rule of thumb.

Unsophisticated price setters are introduced into both sectors because standard New Keynesian models with purely forward looking price setting mechanisms fail to explain the hump shaped responses of sectoral inflations to demand and supply shocks. Introducing this type of backward looking behavior helps to alleviate this problem.

## 3.4 Log-linearization of the Model

In this paper, the equations of the model, which is a general form of the model used by Benigno and Lopez-Salido (2006), are a quite complicated system of stochastic non-linear difference equations. I log-linearize the model around its steady state with zero inflation and study the dynamics of this approximate model.

The relative price charged at time $t$ by firms with new prices of differentiated goods in sector $i$ is denoted by $x_{i,t}^k = p_{i,t}^k / P_{i,t}$, $k$=$f$ for price set by forward looking behavior and $k$=$b$ for price set according to rule of thumb. $x_{i,t} = P_{i,t}/P_t$ denotes the relative price of each sector.

The log-linearized Euler conditions (9) and (10) imply the following IS curve [10]

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(r_t - E_t\hat{\pi}_{t+1} - \hat{\xi}_t^d + E_t\hat{\xi}_{t+1}^d) \tag{17}$$

[10] Variables with hats denote the percentage deviations from the steady state.

The NKPC of the sector $i$ is given by[11]

$$\hat{\pi}_{i,t} = \kappa_{i1}(\hat{Y}_{i,t} - \hat{Y}^n_{i,t}) + \kappa_{i2}\hat{\pi}_{i,t-1} + \kappa_{i3}\hat{\pi}_{i,t+1} + \kappa_{i4}\hat{x}_{i,t} \tag{18}$$

where

$$\kappa_{i1} = \frac{(1-\alpha_i\beta)(\omega^{-1}+\sigma^{-1})(1-\alpha_i)(1-\psi_i)}{(1+\frac{\theta}{\omega})(\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta))}$$

$$\kappa_{i2} = \frac{\psi_i}{\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta)}$$

$$\kappa_{i3} = \frac{\alpha_i\beta}{\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta)}$$

$$\kappa_{i4} = -\frac{(1-\alpha_i\beta)(1+\sigma^{-1})(1-\alpha_i)(1-\psi_i)}{(1+\frac{\theta}{\omega})(\alpha_i+\psi_i(1-\alpha_i+\alpha_i\beta))}$$

$$\hat{Y}^n_{i,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^s_{i,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^d_t$$

The parameter $\omega$ is the elasticity with respect to output of the disutility of supplying production, and $\sigma$ is the elasticity the intertemporal elasticity of substitution.[12] $\hat{Y}^n_{i,t}$ represents a change in the natural rate of output in sector $i$, which is the level of supply that keeps the real marginal cost in sector $i$ constant at the flexible price level.

The measure of inflation persistence is the coefficient of the lagged inflations in the NKPC of the sectors, i.e. $\kappa_{12}$ and $\kappa_{22}$. Note that when $\kappa_{12}$ is zero, the first sector does not display inflation persistence and the model reduces to that of Benigno and Lopez-Salido (2006).

Note that $\lim_{\alpha_i\to 1}\kappa_{i1} = 0$, $\lim_{\psi_i\to 1}\kappa_{i1} = 0$, $\lim_{\alpha_i\to 0}\kappa_{i1} = \psi_i/(1-\psi_i) > 0$ and $\lim_{\psi_i\to 0}\kappa_{i1} = (1-\alpha_i\beta)(1-\alpha_i)/\alpha_1 > 0$. Thus, as the frequency of price change decreases, $\alpha_i$ increases, and the fraction of backward indexing producers increases, the NKPC becomes flatter and sectoral inflation becomes less elastic to the changes in the output gap. In other words, sectoral inflation fails to change inline with the efficient fluctuations in the output

[11]See Appendix A for the derivation.

[12]Here $\sigma = -u''\xi^d C/u'$ and $\omega = -v''\xi^s_i Y_i/v'$ for $i = 1, 2$, evaluated at steady state. Following Aoki (2001), $\omega$ is assumed to be uniform across sectors.

gap. Notice also that $\lim_{\alpha_i \to 1} \kappa_{i2} = \psi_i/(1+\psi_i\beta) < 1$, $\lim_{\psi_i \to 1} \kappa_{i2} = 1/(1+\alpha_i\beta) > 0$, $\lim_{\alpha_i \to 0} \kappa_{i2} = 1$ and $\lim_{\psi_i \to 0} \kappa_{i1} = 0$. Therefore, the degree of persistence in a sector is higher the higher the frequency of price change and the higher the fraction of backward indexing producers. As higher frequency of price change and higher fraction of backward indexing producers imply a higher total fraction of backward looking price setters given by $(1-\alpha)\psi$.

### 3.5 The Loss Function

The central bank is concerned with maximizing the welfare of the households. Following Rotemberg and Woodford (1998, 1999) and Woodford (2003, ch. 6), the welfare measure is the expected utility of the households given by

$$W = E\left\{\sum_{t=0}^{\infty} \beta^t U_t\right\} \tag{19}$$

where

$$U_t = 2u_t(\xi_t^d Y_t/2) - \int_0^1 v(\xi_{1,t}^s y_{1,t}(z))dz - \int_0^1 v(\xi_{2,t}^s y_{2,t}(z))dz \tag{20}$$

Following Aoki (2001), I assume that mass of one households produce for each sector but mass of two consumes each sectors' product. Therefore, in equilibrium the consumption of each good is the half of the production of that good. As in Aoki (2001) and Benigno and Lopez-Salido (2006), I assume that this steady state involves a lump-sum tax, which is set such that the steady state levels of output in both sectors are efficient.

A second order Taylor series approximation of equation (20) around the zero inflation steady state is[13]

$$U_t = -\frac{1}{2}u'\bar{Y}L_t \tag{21}$$

$$L_t = \lambda_1\hat{\pi}_{1,t}^2 + \lambda_2\hat{\pi}_{2,t}^2 + \lambda_3(\hat{Y}_t - \hat{Y}_t^n)^2 + \lambda_4((\hat{Y}_{1,t} - \hat{Y}_{2,t}) - \kappa(\hat{Y}_{1,t}^n - \hat{Y}_{2,t}^n))^2 + \lambda_5(\Delta\hat{\pi}_{1,t})^2 + \lambda_6(\Delta\hat{\pi}_{2,t})^2$$

where $L_t$ is the loss function and the coefficients are given by

[13] See Appendix B for the derivation.

$$\lambda_1 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\alpha_1}{(1-\alpha_1)(1-\alpha_1\beta)}$$

$$\lambda_2 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\alpha_2}{(1-\alpha_2)(1-\alpha_2\beta)}$$

$$\lambda_3 = \sigma^{-1} + \omega^{-1}$$

$$\lambda_4 = \frac{1}{4}(1 + \omega^{-1})$$

$$\lambda_5 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\psi_1}{(1-\alpha_1)(1-\psi_1)(1-\alpha_1\beta)}$$

$$\lambda_6 = \frac{1}{2}(\theta^{-1} + \omega^{-1})\theta^2 \frac{\psi_2}{(1-\alpha_2)(1-\psi_2)(1-\alpha_2\beta)}$$

Notice that, when $\psi_1 = 0$, the loss function simplifies to that of Benigno and Lopez-Salido (2006), where central bank takes into account inflations of both sectors and change in the inflation of the second sector only. The introduction of backward looking price setters makes the deviation of the current inflation from inflation of the previous period a concern of optimal policy, since the relative price of the backward looking price setters are distorted as much as this deviation. Note that, as $\psi$ increases the weight of the deviation of this period's inflation from that of the previous period, $\lambda_5$ or $\lambda_6$, increases. Therefore, for a constant level of price change frequency, as the fraction of backward indexing producers increases, the weights attributed to changes in inflation increases.

Note also that when $\psi_1 = \psi_2 = 0$ the loss function obtained is that of Aoki (2001) and Benigno (2004). Since there exists no backward indexing producers in the economy, deviation in inflation is not a concern of the central bank in itself. Moreover, since the only parameter governing the nominal rigidity in a sector is $\alpha$, once it is equal across sectors, the weights of the sectoral inflations is equal in the loss function. This clearly implies attaching equal weights to sectoral inflations in the optimal inflation targeting rule. These equal weights can only be generated by equal frequency of price change across sectors, which is the sole source of heterogeneity in price setting in the model. Therefore, for purely forward looking models, the weights in the optimal inflation targeting rule is 0.5 *if and only if* $\alpha$ is the same across sectors.

### 3.6 Optimal Inflation Targeting

The model is closed by introducing a strict inflation targeting rule, which has the following form

$$\zeta\pi_{1,t} + (1-\zeta)\pi_{2,t} = 0 \tag{22}$$

where $\zeta$ is the weight that is attributed to the inflation of the first sector. The weight is chosen to maximize the welfare criterion (19) subject to the structural equations of the model ((17) and (18)). Once sectoral asymmetries are introduced, under the inflation targeting regime, the concern of the central bank becomes which inflation to target. Therefore, under the strict inflation targeting rule, the central bank defines the optimal basket, which is determined by optimally choosing the weights that should be attached to each sector.

### 3.7 Model Calibration

The calibrations follow those of Benigno and Lopez-Salido (2002). The discount rate $\beta$ is calibrated as 0.99. I set the parameter $\theta$ equal to 6, which corresponds to a steady-state mark-up of 1.2. The elasticity of substitution in consumption, $\sigma$, is 6 and the elasticity of the disutility of producing the differentiated goods, $\omega$, is 0.6. The average duration of price, which is given by $1/(1-\alpha)$, is calibrated alternatively as 3, 4, 5 and 6 quarters. The fraction of rule of thumb price setters are assumed to be 0.01, 0.3, 0.5 and 0.8 as the fraction of backward indexing producers are estimated to be less than 0.8 for each sector in Massidda (2005) and Leith and Malley (2005). The sectors are assumed to be equal in economic size. The asymmetric supply shocks and the symmetric demand shock follow an AR(1) process of the kind:

$$\mathrm{X}_t = \rho\mathrm{X}_{t-1} + \varepsilon_t$$

where $X_t$ is the vector of shock processes, $X_t$=($\hat{\xi}^s_{1,t}$, $\hat{\xi}^s_{2,t}$, $\hat{\xi}^d_t$), $\rho$ is 0.9 and $\varepsilon_t$ is the vector of independently identified disturbances. The shocks $\hat{\xi}^s_{1,t}$, $\hat{\xi}^s_{2,t}$ and $\hat{\xi}^d_t$ have standard deviations of unity.

# 4 Inflation Persistence and Optimal Inflation Targeting

In this section I analyze the relevance of the inflation persistence for the optimal inflation targeting policy for the two cases: sectors have same degree of inflation persistence and one of the sectors is more persistent than the other sector.

## 4.1 Homogenous Degrees of Inflation Persistence across Sectors

I first calibrate the sectors with a uniform degree of inflation persistence and analyze the relevance of the same reduced form dynamics for the optimal inflation targeting policy. Therefore, the concern of this section is that whether the central bank can ignore the underlying heterogeneities across sectors when they imply same degree of inflation persistence and target the CPI inflation. Since inflation persistence is determined by the two parameters of the price setting mechanism, it is possible to produce same degree of inflation persistence with different combinations of frequency of price change and fraction of backward looking price setters.

Table 1 displays the optimal weight attached to the inflation of the first sector. First column is the duration of prices in the second sector and second column is that of the first sector. For each calibration of the price setting parameters of the second sector and the frequency of the price setting in the first sector, the fraction of backward indexing producers in the first sector is computed consistent with calibrations of other parameters so that the degree of inflation persistence of the first sector is equal to that of the second sector. Note that, CPI targeting is optimal only if the sectors are characterized by exactly the same price setting mechanism. That is, optimal weight is equal to the economic size of each sector, 0.5, when they have the same frequency of price change and fraction of backward looking price setters. When the sectors have heterogenous price setting mechanisms, the optimal weight takes values between 0.20 and 0.77. This implies that even if both sectors have the same degree of inflation persistence, the optimal inflation targeting rule is not CPI targeting.

Since the degree of inflation persistence is the same across sectors, a higher duration of price in one sector is accompanied with a higher fraction of backward looking price setters than the other sector. Therefore, as the duration of price increases in the first sector, the weight attached to its inflation increases.

## 4.2 Heterogenous Degrees of Inflation Persistence across Sectors

The previous section analyzed whether the CPI inflation targeting is the optimal inflation targeting rule under uniform calibration of the inflation persistence. It is shown that even if the sectors have uniform degree of inflation persistence, the differences in the price setting mechanism should be considered when designing the optimal inflation targeting measure. In this section the relevance of higher degree of inflation persistence in one sector for optimal monetary policy is analyzed.

First note that persistence in one sector can be made higher than that of the other sector by assuming a higher total fraction of the backward looking price setters in that sector, namely $(1-\alpha)\psi$. Clearly, this can happen in many ways for different combinations of $\alpha$ and $\psi$.

Benigno and Lopez-Salido (2006) show that for a constant fraction of backward indexing producers, the weight attached to the inflation of the sector increases with the increase in the duration of price. Similarly, keeping the duration of price constant, the weight of the inflation of the sector increases as the fraction of backward price setters increases. In this paper, the calibrations span all possible cases changing both parameters to find out which effect dominates.

For different feasible calibrations of the parameters of the price setting mechanisms of both sectors, the second sector is calibrated to be more persistent than the first sector and the weights in the optimal inflation targeting rule are computed. The first panel of Table 2 displays the optimal weights attached to the inflation of the first sector when the second sector is more persistent than the first sector by 0.1 points. Since the weight in the table is that of the less persistent sector, one would expect that it takes values less than 0.5. However, results show that for some calibrations the optimal weight is higher than the economic size of 0.5 and implies that central bank should pay higher attention to the inflation of the less persistent sector. The weight attached to the inflation of the first sector decreases as the fraction of backward looking price setters in the second sector increases.

The robustness of the result is checked by calibrating the inflation of the second sector to be 0.3 points higher than that of the first sector. The optimal weights are reported in the second panel of Table 2. Results show that for some of the parameter calibrations, although the first sector is significantly less persistent than the first sector, a higher weight is attributed to its inflation in the optimal inflation targeting rule. The difference between the first and second panels of Table 2 is that the weights in the second panel are smaller than the corresponding weights in the first panel. That is, for 3 quarters of duration of prices in both sectors and 0.3 of fraction of backward looking price setters in the second sector, the weight attached to the inflation of the first sector in first panel is 0.461, whereas the corresponding weight in the second panel is 0.410. Therefore, as the gap between the degree of persistence of the sectoral inflations increases, the weight attached to the less persistent sector decreases.

Having shown that the optimal inflation targeting rule is not designed considering the degree of inflation persistence, the next section explores whether the reason behind this fact is the implications related to the slope of the NKPC.

## 4.3 Slope of the NKPC and Optimal Inflation Targeting

As mentioned before, models without inflation persistence suggest that the central bank should attribute a higher weight to the inflation of the sector with a higher duration of price. This is equivalent to the suggestion that stabilization of the inflation of the sector that has a flatter NKPC should be more of a concern for monetary policy. In the model presented in this paper, the slope of the NKPC is not only a function of the frequency of price change but also the fraction of backward looking price setters. Therefore, this section addresses the question whether optimal inflation targeting policy is designed according to the relative slope of the sectoral NKPCs in a generalized model.

Panels of Figure 2 display the optimal weights attached to the inflation of the first sector obtained in the previous sections as a function of the relative slope of the first sector when the differential in inflation persistence across sectors is 0, 0.1 and 0.3 points and fraction of backward looking producers is 0.5.[14] The first panel shows that for a given duration of prices in the second sector as the relative slope increases, the weight attached to the inflation of the first sector decreases. Moreover, when the NKPC of the first sector is flatter than that of the second sector, the optimal weight attached to the inflation of it is higher than 0.5. Similarly, when the NKPC of the first sector is steeper than that of the second sector, a lower weight is attached to its inflation. The second and the third panels of Figure 2 presents Therefore, the weight attached to the inflation of the first sector is a function of the slope of the NKPC but not that of the inflation persistence per se. Even though the second sector is calibrated to be more persistent than the first sector, unless it has a flatter NKPC, stabilization of its inflation becomes less of concern for the central bank.[15] As in the models without inflation persistence, it is optimal to pay higher attention to the inflation of the sector that generates higher real distortions and fails to adjust to the efficient fluctuations in the sectoral output gap.

[14] Relative slope is obtained by dividing the slope of the NKPC of the first sector to that of the second sector.

[15] For some calibrations that are not reported here, the optimal weight is not equal across sectors when they have NKPCs that have about the same slope, which can be produced by different price setting mechanisms. Because of that the slope of the NKPC is not a sufficient summary statistic for the optimal inflation targeting policy design here, unlike in the case of models without inflation persistence.

# 5 Welfare Analysis

Two sector models of Aoki (2001) and Benigno (2004) and the multi-sector model of Eusepi et al. (2009) imply that optimal inflation targeting policy approximates the optimal commitment policy well. Benigno and Lopez-Salido (2006) shows that optimal inflation targeting policy is outperformed by output gap targeting policy for some parameter calibrations. In the following subsections, I report the welfare cost of adopting inflation targeting policy instead of the optimal policy for the two cases where the sectors have same degree of inflation persistence and one of the sectors is more persistent than the other sector. Combining the results, I analyze the relevance of the *level* of inflation persistence in the economy and the *magnitude of the difference* in inflation persistence across sectors for welfare.

## 5.1 Welfare Analysis under Homogenous Degrees of Inflation Persistence across Sectors

For each calibration of the optimal inflation targeting rule in the previous section, Table 1(b) shows the welfare cost of inflation targeting policy instead of optimal policy when sectors have same degree of inflation persistence. The welfare measure is the extra loss as a percentage of optimal loss which is given by the expression:

$$D_{IT} = \frac{E(W_{InflationTargeting}) - E(W_{Optimal})}{E(W_{Optimal})} \times 100 \tag{23}$$

where $W_{InflationTargeting}$ and $W_{Optimal}$ are the welfare loss in terms of steady state consumption under optimal inflation targeting rule and the optimal commitment rule, respectively. When both sectors have the same price setting mechanism, the optimal inflation targeting rule coincides with the optimal commitment rule as is the case in the models without persistence. As far as nonuniform price setting mechanisms are concerned, the welfare loss is less than one percent when both sectors do not display inflation persistence. In other words, when the fraction of backward indexing price setters in the second sector is 0.01 and the degree of inflation persistence in both sectors is close to zero, optimal inflation targeting policy approximates the optimal policy well. However, the welfare cost increases as the fraction of backward indexing producers in the second sector increases. Optimal inflation targeting policy implies an additional welfare loss as high as 7.5% when the fraction of backward indexation is 0.8 and duration of prices is 6 quarters in the second sector.

## 5.2 Welfare Analysis under Heterogenous Degrees of Inflation Persistence across Sectors

In this section, the welfare cost of optimal inflation targeting rule is considered relative to the optimal rule according to the welfare measure (23) for a wide set of feasible calibrations. Persistence of the first sector is calibrated to 0.1 and 0.3 points less than that of the second sector, keeping this difference constant. Therefore, as persistence in the second sector increases that of first sector also increases. Thus, level of inflation persistence in the economy as a whole increases.

The first panel of the Table 3 displays the extra welfare loss resulting from optimal inflation targeting as a percentage of the optimal loss when the degree of inflation persistence in the second sector is 0.1 point higher than that of the first sector. Optimal inflation targeting rule approximates the optimal policy best when the duration of price and the fraction of backward looking price setters in the second sector are lowest. The percentage welfare loss increases as the average duration of prices and the fraction of backward looking price setters increases. That is, keeping the fraction of backward indexing producers in the second sector at 0.3, when duration of prices in the first sector is 3 quarters, the percentage loss increases from 1.96 to 3.16, 4.33 and 5.39 as the duration of prices in the second sector increases from 3 quarters to 4, 5 and 6 quarters. Similarly, keeping the average duration of prices at 3 quarters in both sectors, as the fraction of backward looking price setters in the second sector increases from 0.3 to 0.55 and 0.8, the percentage welfare loss increases from 1.96 to 4.34 and 20.76.

Note that, higher duration of prices and higher fraction of backward looking price setters have different implications for the level of the inflation persistence. Higher duration implies lower degree of inflation persistence, whereas higher fraction of backward looking price setters implies a higher degree of inflation persistence for both sectors. Therefore, it is not possible to draw conclusions regarding the relevance of the *level* of inflation persistence in the economy for the welfare loss resulting from optimal inflation targeting. Instead of the level of inflation persistence in the economy, its source is relevant for welfare.

Second panel of the Table 3 presents the welfare loss when second sector is 0.3 points more persistent than the first sector. Similar to the results in the first panel, optimal inflation targeting policy fails to approximate the optimal commitment rule and the welfare loss does not increase as the level of persistence increases. The welfare loss increases as the fraction of backward indexing producers in the second sector increases and implies almost as high as twice the welfare loss under optimal rule when $D_2 = 6$, $D_1 = 5$, $\psi_2 = 0.8$ and $\psi_1$ is 0.19.

Another interesting finding is that, for a given calibration of frequency of price change in both sectors and fraction of backward indexing price setters in the second sector, as the difference in the degree of inflation persistence across sectors increases the welfare loss also increases. To illustrate, for $D_2 = 4$ and $\psi_2 = 0.8$ the degree of inflation persistence in the second sector is 0.5. Keeping $D_1 = 3$ and calibrating the first sector so that the persistence is equal to 0.5, 0.4 and 0.2 implies additional welfare losses of 3.153, 24.70 and 56.35 percent, respectively. Therefore, it appears that as the degree of inflation persistence differential across sectors increases, optimal inflation targeting rule implies higher welfare losses when compared to the optimal commitment policy.

## 5.3 Impulse Responses to Negative Supply Shock

To gain the intuition of these results, Figures 3 to 5 display the responses of the sectoral inflations and output gaps, which is the deviation of the actual output from its natural rate, to a negative supply shock to the second sector. Following a negative supply shock, the production in the second sector decreases and inflation of the second sector increases. As the reduction in the production is less than the change in the natural rate of output of the second sector, output gap increases in the second sector. Due to income effect of the higher price of the second sector's product, demand for both sectors' products decreases. Decrease in the demand for the first sector good decreases the inflation of the first sector and produces negative output gap since the natural rate of output is constant for the first sector.

The implications of the magnitude of the differential in sectoral inflation persistence can be analyzed by comparing the responses in the three panels of the Figures 3 to 5. The first panel of Figure 3 displays the impulse responses when both sectors have same the degree of inflation persistence and the second and third panels display the responses when the second sector is 0.1 points and 0.3 points more persistent than the first sector, respectively. In the first panel, responses under optimal inflation targeting rule approximates that of the optimal commitment rule well. However, as the difference between the persistence of sectoral inflations increases, the responses under optimal inflation targeting rule diverges from those of the optimal rule. Therefore, the welfare loss relative to the optimal loss increases as the degree of inflation persistence differential increases.

Similar to Figure 3, Figures 4 and 5 show that as the sectors diverge in terms of degree of inflation persistence, optimal inflation targeting regime fails to approximate the optimal commitment rule and implies significant differences in terms of responses of sectoral inflations and output gaps.

As far as the implications of higher level of inflation persistence are concerned, the analysis can be done by comparing each impulse response across figures. The relevance of the level of inflation persistence in the overall economy can be analyzed by comparing the impulse responses in Figure 4 to those of Figures 3 and 5. Inflation persistence in the second sector is 0.43, 0.4 and 0.52 in Figure 3, Figure 4 and Figure 5 , respectively. The source of higher persistence in Figure 3 is higher frequency of price change, whereas inflation persistence is higher in Figure 5 as the fraction of backward looking price setters is higher than the one in Figure 3. The impulse responses in all three panels of the Figure 3 provide slightly better approximations to the optimal responses than the ones in the less persistent economy, namely Figure 4. Therefore, when the source of higher persistence in the economy is the increased frequency of price change, depending on the parameter calibrations, a higher persistence may imply a higher or lower additional cost of adopting optimal inflation targeting.

However, impulse responses in the Figure 5 deviates significantly more from the deviation observed between the impulse responses in Figure 4. Therefore, if the higher persistence is resulting from a higher fraction of backward looking price setters, optimal inflation targeting policy produces impulse responses that deviates more from the those under the optimal rule when inflation persistence in the economy is higher.

# 6 Conclusions

In this paper, I extend the models existing in the literature in order to introduce inflation persistence into two sectors and analyze the relevance of the inflation persistence for the optimal inflation targeting policy design. I show that even if the sectoral inflations have the same degree of persistence, optimal inflation targeting policy attaches different weights to them unless they are characterized by equal frequency of price change and fraction of backward looking price setters.

The main contribution of this paper is the finding that the optimal inflation targeting rule does not always attach a higher weight to the inflation of the more persistent sector. Rather, the optimal weight is determined by the relative flatness of the New Keynesian Phillips curve. That is, the weight attached to a sector is higher when the sector has a flatter NKPC than the other sector.

The welfare implications of the optimal inflation targeting policy are analyzed using a loss function that is produced as a second order approximation to the utility function. The results show that in contrast to the models without inflation persistence, in this model, optimal inflation targeting policy fails to approximate the optimal commitment rule. I differentiate between the sources of higher inflation persistence in the economy and show that if the persistence is resulting from a higher fraction of backward looking price setters, the performance the optimal inflation targeting rule decreases. However, if the source of higher inflation persistence is lower duration of prices, the extra welfare loss implied by optimal inflation targeting decreases as inflation persistence increases.

As far as the relevance of the magnitude of the heterogeneity is concerned, as the difference between the degree of inflation persistence increases, the welfare loss implied by the optimal inflation targeting policy increases. Therefore, optimal inflation targeting policy may not be an appropriate policy when the inflation persistence differential across sectors is high.

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# Appendix

## A. Derivation of the New Keynesian Phillips Curve

Equation (11) can be written in terms of stationary variables as follows:

$$E_t \sum_{k=0}^{\infty} (\alpha_i \beta)^k \Omega'_{i,t+k} x^f_{i,t} = E_t \sum_{k=0}^{\infty} (\alpha_i \beta)^k \Omega'_{i,t+k} \frac{\theta}{\theta - 1} s_{i,t+k} \tag{A.1}$$

where $x^f_{i,t} = P^f_{i,t}/P_t$, $x_{i,t} = P_{i,t}/P_t$, $s_{i,t+k} = S_{i,t+k}/P_{t+k}$, $\pi_{i,t+s} = P_{t+s}/P_{t+s-1}$ and $\pi_{i,t+s} = P_{i,t+s}/P_{i,t+s-1}$

$$\Omega'_{i,t+k} = \frac{\Omega_{i,t+k}}{P_{t+k}} = \frac{\xi^d_{t+k} u'(\xi^d_{t+k} C_{t+k})}{\xi^d_t u'(\xi^d_t C_t)} x^{-1}_{i,t} (x^f_{i,t} \prod_{s=1}^{k} \pi^{-1}_{i,t+s})^{-\theta} C_{t+k} \tag{A.2}$$

Log-linearization of the equation (A.1)is

$$\sum_{k=0}^{\infty} (\alpha_i \beta)^k \hat{x}^f_{i,t} + \hat{x}_{i,t} - E_t \sum_{s=0}^{k} \pi_{t+1+s} = \sum_{k=0}^{\infty} (\alpha_i \beta)^k E_t \hat{s}_{t+k} \tag{A.3}$$

This can be written as

$$(\hat{x}^f_{i,t} + \hat{x}_{i,t}) \sum_{k=0}^{\infty} (\alpha_i \beta)^k = \sum_{k=0}^{\infty} (\alpha_i \beta)^k E_t (\hat{s}_{t+k} + \sum_{s=0}^{k} \pi_{t+1+s}) \tag{A.4}$$

This equation can be recursively written as follows

$$\hat{x}^f_{i,t} + \hat{x}_{i,t} = (\alpha_i \beta)(\hat{x}^f_{i,t} + \hat{x}_{i,t}) + (1 - \alpha_i \beta) \left\{ \hat{s}_t + \frac{1}{1 - \alpha_i \beta} \pi_{t+1} \right\} \tag{A.5}$$

$\hat{s}_{t+k}$ is obtained by loglinearizing the equation (13) in the text

$$\hat{s}_{t+k} = \hat{\xi}^s_{i,t+k}(1 + \frac{1}{\omega}) + \frac{1}{\omega} \hat{y}(z)_{i,t+k} - \hat{\xi}^d_{t+k}(1 - \frac{1}{\sigma}) + \frac{1}{\sigma} \hat{Y}_{t+k} \tag{A.6}$$

where loglinearizing the demand condition (7)

$$\hat{y}(z)_{i,t+k} = -\hat{x}_{i,t+k} - \theta \hat{x}^f_t + \theta \sum_{s=0}^{k} \pi_{t+1+s} + \hat{Y}_{t+k} \tag{A.7}$$

plugging this back to (A.6)

$$\hat{s}_{t+k} = (\frac{1}{\omega} + \frac{1}{\sigma})\hat{Y}_{t+k} + \hat{\xi}^s_{i,t+k}(1 + \frac{1}{\omega}) - \frac{1}{\omega}(\hat{x}_{i,t+k} - \theta\hat{x}^f_t + \theta\sum_{s=0}^{k}\pi_{t+1+s}) - \hat{\xi}^d_{t+k}(1 - \frac{1}{\sigma}) \quad (A.8)$$

$$= (\frac{1}{\omega} + \frac{1}{\sigma})(\hat{Y}_{t+k} - \hat{Y}^n_{i,t+k}) - \frac{1}{\omega}(\hat{x}_{i,t+k} - \theta\hat{x}^f_t + \theta\sum_{s=0}^{k}\pi_{t+1+s})$$

where

$$\hat{Y}^n_{i,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^s_{i,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}^d_t$$

Next, loglinearization of price indices (14)-(16) and the demand condition (5) produce the following identities:

$$\hat{x}^b_{i,t} = \hat{x}^*_{t-1} - \pi_{i,t} + \pi_{i,t-1} \quad (A.9)$$

$$\hat{p}^*_{t-1} = (1-\psi_i)\hat{x}^f_t + \psi_t\hat{x}^b_t \quad (A.10)$$

$$\pi_t = \frac{\alpha}{1-\alpha}((1-\psi_i)\hat{x}^f_t + \psi\hat{x}^b_t)) \quad (A.11)$$

$$\hat{Y}_{i,t} = -\hat{x}_{i,t} + \hat{Y}_t \quad (A.12)$$

Combining the equations (A.8)-(A.12)produce the New Keynesian Phillips curve (18) in the text.

## B.Derivation of the Loss Function

In this section I derive the utility based loss function for the model. The period $t$ welfare of the economy is the average utility over the continuum of households is given by the expression:

$$W_t = 2U_t(\xi^d_t Y_t/2) - \int_0^1 \upsilon(\xi^s_{1,t}y_{1,t}(z))dz - \int_0^1 \upsilon(\xi^s_{2,t}y_{2,t}(z))dz \quad (B.1)$$

First, a first order Taylor series approximation is taken for the first term of the equation

$$\begin{aligned} U_t(\xi^d_t Y_t/2) &= U_{Y_1}(Y_{1,t} - \bar{Y_1}) + U_{Y_2}(Y_{2,t} - \bar{Y_2}) + \frac{1}{2}U_{Y_1Y_1}(Y_{1,t} - \bar{Y_1})^2 \quad (B.2) \\ &+\frac{1}{2}U_{Y_2Y_2}(Y_{2,t} - \bar{Y_2})^2 + U_{Y_1Y_2}(Y_{1,t} - \bar{Y_1})(Y_{2,t} - \bar{Y_2}) \\ &+U_{\xi^d Y_1}(\xi^d_t - \bar{\xi^d})(Y_{1,t} - \bar{Y_1}) + U_{\xi^d Y_2}(\xi^d_t - \bar{\xi^d})(Y_{2,t} - \bar{Y_2}) + t.i.p + O(3) \end{aligned}$$

where $U_{Y_i} \equiv \partial U/\partial Y_i$ and $U_{Y_iY_i} \equiv \partial^2 U/\partial Y_i{}^2$. $\bar{Y_1} and \bar{Y_2}$ are the optimal equilibrium levels of output of sectors and $\bar{\xi}^d$ is the steady state value of the identical demand shock. Using the fact that $Y_i = \bar{Y_{i,t}}(1+\hat{Y_{i,t}}+\frac{1}{2}\hat{Y_{i,t}}^2)+O(3)$, (2) can be written as:

$$U_t(\xi_t^d Y_t/2) = U_{Y_1}\bar{Y}_1\hat{Y}_{1,t}+U_{Y_2}\bar{Y}_2\hat{Y}_{2,t}+\frac{1}{2}(U_{Y_1}\bar{Y}_1+U_{Y_1Y_1}\bar{Y}_1^2)\hat{Y}_{1,t}^2 + \frac{1}{2}(U_{Y_2}\bar{Y}_2+U_{Y_2Y_2}\bar{Y}_2^2)\hat{Y}_{2,t}^2+U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2\hat{Y}_{1,t}\hat{Y}_{2,t} + U_{\xi^dY_1}\bar{\xi^d}\bar{Y}_1\hat{\xi}_t^d\hat{Y}_{1,t}+U_{\xi^dY_2}\bar{\xi^d}\bar{Y}_2\hat{\xi^d}_t\hat{Y}_{2,t}+t.i.p+O(3) \tag{B.3}$$

where $\hat{Y}_{i,t}\equiv\log(Y_{i,t}/\bar{Y})$. A second order approximation to the second and the third terms of (1) and similar manipulations give the following expression

$$\upsilon(\xi_{i,t}^s y_{i,t}(z)) = \upsilon_{y_i}\bar{Y}_i\hat{y}_{i,t}(z)+\frac{1}{2}(\upsilon_{y_i}\bar{Y}_i+\upsilon_{y_iy_i}\bar{Y}_i^2)\hat{y}_{i,t}(z)^2 + \upsilon_{\xi_i^s y_i}\bar{\xi}_i^s\bar{Y}_i\hat{y}_{i,t}(z)\hat{\xi}_{i,t}^s+t.i.p+O(3) \tag{B.4}$$

where $\upsilon_{y_i}\equiv\partial\upsilon/\partial y_i$, $\upsilon_{y_iy_i}\equiv\partial^2\upsilon/\partial {y_i}^2$ and $\hat{y}_t(z)\equiv\log(y_t(z)/\bar{Y})$. $\bar{\xi}_i^s$ is the steady state value of sector specific supply shock. Integrating (4) over $z$

$$\int_0^1\upsilon(\xi_{i,t}^s y_{i,t}(z))dz = \upsilon_{y_i}\bar{Y}_iE_z[\hat{y}_{i,t}(z)]+\frac{1}{2}(\upsilon_{y_i}\bar{Y}_i+\upsilon_{y_iy_i}\bar{Y}_i^2)var_z[\hat{y}_{i,t}(z)] + \frac{1}{2}(\upsilon_{y_i}\bar{Y}_i+\upsilon_{y_iy_i}\bar{Y}_i^2)E_z[\hat{y}_{i,t}(z)]^2 + \upsilon_{\xi^s y_i}\bar{\xi^s}\bar{Y}_i\hat{\xi}_{i,t}^sE_z[\hat{y}_{i,t}(z)]+t.i.p+O(3) \tag{B.5}$$

Here $E_z[\cdot]$ and $var_z[\cdot]$ represent the population average and variance of the outputs of the producers, respectively. A Taylor series expansion of the Dixit-Stiglitz aggregator (3) is given by

$$\hat{Y}_{i,t} = E_z[\hat{y}_{i,t}(z)]+\frac{1}{2}\frac{\theta-1}{\theta}var_z[\hat{y}_{i,t}(z)]+O(3) \tag{B.6}$$

Solving for $E_z[\cdot]$ and plugging it back to (5) yields

$$\int_0^1\upsilon(\xi_{i,t}^s y_{i,t}(z))dz = \upsilon_{y_i}\bar{Y}_i\hat{Y}_{i,t}+\frac{1}{2}(\upsilon_{y_i}\bar{Y}_i+\upsilon_{y_iy_i}\bar{Y}_i^2)\hat{Y}_{i,t}^2 + \frac{1}{2}(\theta^{-1}\upsilon_{y_i}\bar{Y}_i+\upsilon_{y_iy_i}\bar{Y}_i^2)var_z[\hat{y}_{i,t}(z)] + \upsilon_{\xi^s y_i}\bar{\xi^s}\bar{Y}_i\hat{\xi}_{i,t}^s\hat{Y}_{i,t}^2+t.i.p+O(3) \tag{B.7}$$

Substituting (5) and (7) into welfare equation (1)gives

$$
\begin{aligned}
W_t \quad = \quad & \frac{1}{2}(2U_{Y_1Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1)\bar{Y}_1{}^2\hat{Y}_{1,t}^2 + \frac{1}{2}(2U_{Y_2Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2)\bar{Y}_2{}^2\hat{Y}_{2,t}^2 \\
& +(2U_{\xi^dY_1}\bar{\xi^d}\bar{Y}_1\hat{\xi}_t^d - \upsilon_{\xi_1^sy_1}\bar{\xi}_1^s\bar{Y}_1\hat{\xi}_{1,t}^s)\hat{Y}_{1,t} \\
& +(2U_{\xi^dY_2}\bar{\xi^d}\bar{Y}_2\hat{\xi}_t^d - \upsilon_{\xi_2^sy_2}\bar{\xi}_2^s\bar{Y}_2\hat{\xi}_{2,t}^s)\hat{Y}_{2,t} \\
& +2U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{2}(2\theta^{-1}U_{Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1^2)var_z[\hat{y}_{1,t}(z)] \\
& -\frac{1}{2}(2\theta^{-1}U_{Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2^2)var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3).
\end{aligned}
\tag{B.8}
$$

Notice that $2U_{Y_i} = \upsilon_{y_i}$ since the approximation is around efficient steady state and linear terms cancel out.

In order to write (8) in terms of natural rates, the following relationships are

$$
\begin{aligned}
\upsilon_{\xi_i^sy_i} &= \upsilon'(\frac{1}{\omega} + 1) \\
U_{\xi^dY_i} &= \frac{1}{4}(-\frac{1}{\sigma} + 1)U'\frac{\bar{Y}}{\bar{Y}_i}
\end{aligned}
$$

where $\omega = -\upsilon''\bar{\xi}_i^s\bar{Y}_i/\upsilon'$ and $\sigma = -U''\bar{\xi^d}\bar{C}/U'$. Therefore, the third and the fourth terms in (8) can be written as:

$$
(2U_{\xi^dY_1}\bar{\xi^d}\bar{Y}_1\hat{\xi}_t^d - \upsilon_{\xi_1^sy_1}\bar{\xi}_1^s\bar{Y}_1\hat{\xi}_{1,t}^s)\hat{Y}_{1,t} = (\frac{1}{\omega} + \frac{1}{\sigma})2U_{Y_i}\bar{Y}_i\hat{Y}_{i,t}^n\hat{Y}_{i,t} \tag{B.9}
$$

where

$$
\hat{Y}_{i,t}^n \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{\xi}_{i,t}^s - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{\xi}_t^d
$$

is the natural rate of output for $i = 1, 2$. Substituting (11) back to (8)gives:

$$
\begin{aligned}
W_t \quad = \quad & \frac{1}{2}(2U_{Y_1Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1)\bar{Y}_1{}^2\hat{Y}_{1,t}^2 + \frac{1}{2}(2U_{Y_2Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2)\bar{Y}_2{}^2\hat{Y}_{2,t}^2 \\
& +2(\frac{1}{\omega} + \frac{1}{\sigma})U_{Y_1}\bar{Y}_1\hat{Y}_{1,t}^n\hat{Y}_{1,t} + 2(\frac{1}{\omega} + \frac{1}{\sigma})U_{Y_2}\bar{Y}_2\hat{Y}_{2,t}^n\hat{Y}_{2,t} \\
& +2U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{2}(2\theta^{-1}U_{Y_1}\bar{Y}_1 - \upsilon_{y_1y_1}\bar{Y}_1^2)var_z[\hat{y}_{1,t}(z)] \\
& -\frac{1}{2}(2\theta^{-1}U_{Y_2}\bar{Y}_2 - \upsilon_{y_2y_2}\bar{Y}_2^2)var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3).
\end{aligned}
\tag{B.10}
$$

The variance of the sectoral outputs can be written as variance of the prices of the differentiated of products in each sector by utilizing the demand condition (7)as follows:

$$var_z[\hat{y}_{i,t}(z)] = \theta^2 var_z[logp_{i,t}(z)] + O(3) \tag{B.11}$$

By the definition of variance

$$var_z[logp_{i,t}(z)] = E_z[logp_{i,t}(z)^2] - (E_z[logp_{i,t}(z)])^2$$

Since $E_z[logp_{i,t-1}(z)]$ is a constant in terms of $z$ this can also be written as follows, which gives the price dispersion in each sector:

$$\begin{aligned} var_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]] &= E_z[(logp_{i,t}(z) - E_z[logp_{i,t-1}])^2] \\ &\quad -(E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]])^2 \end{aligned} \tag{B.12}$$

Using the evolution of the aggregate price level of each sector the square root of second term in (12)can be written as:

$$\begin{aligned} E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]] &= \alpha_i E_z[logp_{i,t-1} - E_z[logp_{i,t-1}]] \\ &\quad +(1-\alpha_i)(1-\psi_i)(logp^f_{i,t} - E_z[logp_{i,t-1}]) \\ &\quad +(1-\alpha_i)\psi_i(logp^b_{i,t} - E_z[logp_{i,t-1}]) \\ &= (1-\alpha_i)(logp^*_{i,t} - E_z[logp_{i,t-1}]) \end{aligned} \tag{B.13}$$

The first term in equation (12) can be rewritten as:

$$\begin{aligned} E_z[logp_{i,t}(z) - E_z[logp_{i,t-1}]]^2 &= \alpha_i E_z[logp_{i,t-1}(z) - E_z[logp_{i,t-1}]]^2 + \\ &\quad +(1-\alpha_i)(1-\psi_i)(logp^f_{i,t} - E_z[logp_{i,t-1}])^2 \\ &\quad +(1-\alpha_i)\psi_i(logp^b_{i,t} - E_z[logp_{i,t-1}])^2 \end{aligned} \tag{B.14}$$

Using

$$
\begin{aligned}
p^b_{i,t} &= p^*_{i,t-1}\pi_{i,t-1} \\
p^*_{i,t-1} &= (p^f_{i,t-1})^{1-\psi_i}(p^b_{i,t-1})^{\psi_i} \\
E_z[logp_{i,t}(z)] &= logP_{i,t} + O(2)
\end{aligned}
$$

The second and the third terms in (14) can be further expressed as

$$
\begin{aligned}
logp^b_{i,t} - E_z[logp_{i,t-1}] \quad &= logp^*_{i,t-1} + \pi_{i,t-1} + logP_{i,t-1} + O(2) \\
&= logp^*_{i,t-1} + logP_{i,t-2} + O(2)
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
logp^f_{i,t} - E_z[logp_{i,t-1}] &= \frac{1}{(1-\psi_i)}(logp^*_{i,t} - \psi_i(logp^*_{i,t-1} + \pi_{i,t-1})) - logP_{i,t-1} + O(2) \\
&= \frac{1}{(1-\psi_i)}(logp^*_{i,t} - logP_{i,t-1} + \psi_i(p^*_{i,t-1} + logP_{i,t-2})) + O(2)
\end{aligned}
$$

Finally by plugging (15)and (16) back to (14)

$$
\begin{aligned}
var_z[logp_{i,t}(z)] &= \alpha_i var_z[logp_{i,t-1}(z)] + \frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} \\
&+ \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t} + O(3)
\end{aligned}
\tag{B.16}
$$

Solving this equation forward, starting with an initial variance of the prices, $var_z[logp_{i,-1}(z)]$, which is predetermined and independent of policy at time $t$

$$
\begin{aligned}
var_z[logp_{i,t}(z)] &= \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t}\right) \\
&+ \alpha_i^t var_z[logp_{i,-1}(z)] + O(3) \\
&= \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi^2_{i,t} + \frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi^2_{i,t}\right) + t.i.p. + O(3)
\end{aligned}
\tag{B.17}
$$

The equation (10) can be further developed using the following relations

$$\begin{aligned}
U_{Y_1}\bar{Y}_1 &= U_{Y_2}\bar{Y}_2 = \frac{1}{4}U'\bar{Y} \\
U_{Y_1Y_1}\bar{Y}_1^2 &= U_{Y_2Y_2}\bar{Y}_2^2 = -\frac{1}{4}U'\bar{Y}[\frac{1}{2\sigma}+\frac{1}{2}] \\
U_{Y_1Y_2}\bar{Y}_1\bar{Y}_2 &= \frac{1}{8}U'\bar{Y}(1-\frac{1}{\sigma}) \\
\upsilon_{y_1y_1}\bar{Y}_1^2 &= \upsilon_{y_2y_2}\bar{Y}_2^2 = \frac{1}{2\omega}U'\bar{Y}
\end{aligned}$$

Substituting these identities back to (10) and using the efficient steady state condition $2U_{Y_i} = \upsilon_{y_i}$ yields

$$\begin{aligned}
W_t &= -\frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma}+\frac{1}{2}+\frac{1}{\omega})\hat{Y}_{1,t}^2 - \frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma}+\frac{1}{2}+\frac{1}{\omega})\hat{Y}_{2,t}^2 \\
&+\frac{1}{2}(\frac{1}{\omega}+\frac{1}{\sigma})U'\bar{Y}\hat{Y}_{1,t}^n\hat{Y}_{1,t} + \frac{1}{2}(\frac{1}{\omega}+\frac{1}{\sigma})U'\bar{Y}\hat{Y}_{2,t}^n\hat{Y}_{2,t} \\
&+\frac{1}{4}(-\frac{1}{\sigma}+1)U'\bar{Y}\hat{Y}_{1,t}\hat{Y}_{2,t} - \frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{1,t}(z)] \\
&-\frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3).
\end{aligned} \tag{B.18}$$

Adding and subtracting $\frac{1}{4}U'\bar{Y}(\frac{1}{2\sigma}+\frac{1}{2}+\frac{1}{\omega})\hat{Y}_{1,t}\hat{Y}_{2,t}$ and arranging gives

$$\begin{aligned}
W_t &= -\frac{1}{2}U'\bar{Y}\Phi_t - \frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{1,t}(z)] \\
&-\frac{1}{4}(\frac{1}{\theta}+\frac{1}{\omega})U'\bar{Y}var_z[\hat{y}_{2,t}(z)] + t.i.p. + O(3)
\end{aligned} \tag{B.19}$$

where

$$\begin{aligned}
\Phi_t &= (\frac{1}{\sigma}+\frac{1}{\omega})(\frac{1}{4}\hat{Y}_{1,t}^2+\frac{1}{4}\hat{Y}_{2,t}^2+\frac{1}{2}\hat{Y}_{1,t}\hat{Y}_{2,t}) \\
&-2(\frac{1}{\sigma}+\frac{1}{\omega})(\frac{1}{2}\hat{Y}_{1,t}^n\hat{Y}_{1,t}+\frac{1}{2}\hat{Y}_{2,t}^n\hat{Y}_{2,t}) \\
&+\frac{1}{4}(1+\frac{1}{\omega})(\hat{Y}_{1,t}^2+\hat{Y}_{2,t}^2-2\hat{Y}_{1,t}\hat{Y}_{2,t})
\end{aligned} \tag{B.20}$$

In order to rewrite $\Phi_t$ the following identities are used:

$$\begin{aligned}
\hat{Y}_t &= \frac{1}{2}\hat{Y}_{1,t}+\frac{1}{2}\hat{Y}_{2,t} \\
\hat{Y}_t^n &= \frac{1}{2}\hat{Y}_{1,t}^n+\frac{1}{2}\hat{Y}_{2,t}^n
\end{aligned}$$

Substitution of these together with adding and subtracting $\hat{Y}_t^n$ and (19) becomes

$$\begin{aligned} W_t &= -\frac{1}{2}U'\bar{Y}\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}^2-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\} \quad \text{(B.21)} \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})(var_z[\hat{y}_{1,t}(z)]+var_z[\hat{y}_{2,t}(z)])+t.i.p.+O(3) \end{aligned}$$

The discounted present value of these terms is obtained as follows

$$\begin{aligned} W &= E_0\sum_{t=0}^{\infty}\beta^t W_t \quad \text{(B.22)} \\ &= -\frac{1}{2}U'\bar{Y}\ E_0\sum_{t=0}^{\infty}\beta^t\left\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})(\hat{Y}_{1,t}^2-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\right\} \\ &- \frac{1}{2}U'\bar{Y}\ E_0\sum_{t=0}^{\infty}\beta^t\left\{(\frac{1}{\theta}+\frac{1}{\omega})(var_z[\hat{y}_{1,t}(z)]+var_z[\hat{y}_{2,t}(z)])\right\}+t.i.p.+O(3) \end{aligned}$$

The last term is

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t var_z[\hat{y}_{i,t}(z)] &= \theta^2\sum_{t=0}^{\infty}\beta^t\sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha_i}{1-\alpha_i}\pi_{i,t}^2+\frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi_{i,t}^2\right)+t.i.p.+O(3) \\ &= \frac{\theta^2}{1-\alpha_i\beta}\sum_{t=0}^{\infty}\beta^t\left(\frac{\alpha_i}{1-\alpha_i}\pi_{i,t}^2+\frac{\psi_i}{(1-\alpha_i)(1-\psi_i)}\Delta\pi_{i,t}^2\right) \quad \text{(B.23)} \end{aligned}$$

Substituting this back:

$$\begin{aligned} W_t &= -\frac{1}{2}U'\bar{Y}\{(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}^2-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^2-\hat{Y}_{2,t}))^2\} \quad \text{(B.24)} \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}(\frac{\alpha_1}{1-\alpha_1}\pi_{1,t}^2+\frac{\psi_1}{(1-\alpha_1)(1-\psi_1)}\Delta\pi_{1,t}^2) \\ &\quad -\frac{1}{4}U'\bar{Y}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}(\frac{\alpha_2}{1-\alpha_2}\pi_{2,t}^2+\frac{\psi_2}{(1-\alpha_2)(1-\psi_2)}\Delta\pi_{2,t}^2)+t.i.p.+O(3) \end{aligned}$$

Define central bank loss function $L_t$ as $W_t=-\frac{1}{2}U'\bar{Y}L_t$. Then rearranging (24) yields

$$\begin{aligned} L_t &= \frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}\frac{\alpha_1}{1-\alpha_1}\pi_{1,t}^2+\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}\frac{\alpha_2}{1-\alpha_2}\pi_{2,t}^2 \quad \text{(B.25)} \\ &\quad +(\frac{1}{\sigma}+\frac{1}{\omega})(\hat{Y}_t-\hat{Y}_t^n)^2+\frac{1}{4}(1+\frac{1}{\omega})((\hat{Y}_{1,t}-\hat{Y}_{2,t})-\kappa(\hat{Y}_{1,t}^n-\hat{Y}_{2,t}^n))^2 \\ &\quad +\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_1\beta}\frac{\psi_1}{(1-\alpha_1)(1-\psi_1)}\Delta\pi_{1,t}^2+\frac{1}{2}(\frac{1}{\theta}+\frac{1}{\omega})\frac{\theta^2}{1-\alpha_2\beta}\frac{\psi_2}{(1-\alpha_2)(1-\psi_2)}\Delta\pi_{2,t}^2 \end{aligned}$$

# Tables

Table 1(a): Optimal Weights attached to the Inflation of the First Sector under uniform Degree of Inflation Persistence across Sectors

| | | $\psi_2$ | | | |
|---|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.01 | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 0.500 | 0.500 | 0.500 | |
| | 4 | 0.622 | 0.622 | 0.628 | |
| | 5 | 0.704 | 0.705 | 0.714 | |
| | 6 | 0.762 | 0.763 | 0.773 | |
| 4 | 3 | 0.378 | 0.378 | 0.375 | 0.343 |
| | 4 | 0.500 | 0.500 | 0.500 | 0.500 |
| | 5 | 0.591 | 0.591 | 0.594 | |
| | 6 | 0.661 | 0.661 | 0.665 | |
| 5 | 3 | 0.296 | 0.296 | 0.292 | 0.257 |
| | 4 | 0.409 | 0.409 | 0.407 | 0.388 |
| | 5 | 0.500 | 0.500 | 0.500 | 0.500 |
| | 6 | 0.574 | 0.574 | 0.576 | |
| 6 | 3 | 0.238 | 0.238 | 0.233 | 0.201 |
| | 4 | 0.339 | 0.339 | 0.336 | 0.312 |
| | 5 | 0.426 | 0.426 | 0.425 | 0.412 |
| | 6 | 0.500 | 0.500 | 0.500 | 0.500 |

$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The values are $\zeta$ in the optimal inflation targeting rule (22).

Table 1(b): Welfare Cost of Inflation Targeting under uniform Degree of Inflation Persistence across Sectors

| | | $\psi_2$ | | | |
|---|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.01 | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 0.000 | 0.000 | 0.000 | |
| | 4 | 0.105 | 0.208 | 0.485 | |
| | 5 | 0.326 | 0.651 | 1.529 | |
| | 6 | 0.596 | 1.178 | 2.750 | |
| 4 | 3 | 0.104 | 0.186 | 0.369 | 3.153 |
| | 4 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 5 | 0.071 | 0.121 | 0.236 | |
| | 6 | 0.242 | 0.405 | 0.773 | |
| 5 | 3 | 0.324 | 0.554 | 1.022 | 5.622 |
| | 4 | 0.071 | 0.116 | 0.209 | 1.239 |
| | 5 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 6 | 0.054 | 0.082 | 0.143 | |
| 6 | 3 | 0.595 | 0.974 | 1.719 | 7.532 |
| | 4 | 0.242 | 0.375 | 0.640 | 2.922 |
| | 5 | 0.054 | 0.082 | 0.135 | 0.624 |
| | 6 | 0.000 | 0.000 | 0.000 | 0.000 |

$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The welfare loss is computed according to (23).

Table 2(a): Optimal Weights attached to the Inflation of the First Sector when Second Sector's Inflation is 0.1 points higher

| | | $\psi_2$ | | |
|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 0.461 | 0.432 | 0.301 |
| | 4 | 0.582 | 0.552 | 0.417 |
| | 5 | 0.666 | 0.638 | 0.510 |
| | 6 | 0.728 | 0.702 | 0.585 |
| 4 | 3 | 0.344 | 0.319 | 0.212 |
| | 4 | 0.459 | 0.430 | 0.297 |
| | 5 | 0.549 | 0.517 | 0.372 |
| | 6 | 0.619 | 0.587 | 0.439 |
| 5 | 3 | 0.267 | 0.247 | 0.163 |
| | 4 | 0.371 | 0.344 | 0.228 |
| | 5 | 0.457 | 0.425 | 0.286 |
| | 6 | 0.528 | 0.495 | 0.341 |
| 6 | 3 | 0.213 | 0.196 | 0.131 |
| | 4 | 0.305 | 0.281 | 0.182 |
| | 5 | 0.385 | 0.355 | 0.228 |
| | 6 | 0.454 | 0.420 | 0.273 |

$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The values are $\zeta$ in the optimal inflation targeting rule (22).

Table 2(b): Optimal Weights attached to the Inflation of the First Sector when Second Sector's Inflation is 0.3 points higher

| | | $\psi_2$ | | |
|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 0.410 | 0.354 | 0.094 |
| | 4 | 0.527 | 0.458 | 0.137 |
| | 5 | 0.611 | 0.537 | 0.185 |
| | 6 | 0.674 | 0.599 | 0.235 |
| 4 | 3 | | 0.261 | 0.057 |
| | 4 | | 0.347 | 0.078 |
| | 5 | | 0.414 | 0.104 |
| | 6 | | 0.471 | 0.133 |
| 5 | 3 | | 0.205 | 0.039 |
| | 4 | | 0.277 | 0.051 |
| | 5 | | 0.331 | 0.068 |
| | 6 | | 0.376 | 0.086 |
| 6 | 3 | | 0.167 | 0.180 |
| | 4 | | 0.230 | 0.324 |
| | 5 | | 0.273 | 0.453 |
| | 6 | | 0.308 | 0.060 |

$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The values are $\zeta$ in the optimal inflation targeting rule (22).

Table 3(a): Welfare Cost of Inflation Targeting when Second Sector's Inflation is 0.1 points higher

| | | $\psi_2$ | | |
|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 1.96 | 4.34 | 20.76 |
| | 4 | 1.52 | 3.41 | 15.67 |
| | 5 | 1.17 | 2.62 | 11.80 |
| | 6 | 0.93 | 2.00 | 8.96 |
| 4 | 3 | 3.16 | 6.46 | 24.70 |
| | 4 | 2.62 | 5.59 | 21.72 |
| | 5 | 2.18 | 4.78 | 18.87 |
| | 6 | 1.85 | 4.09 | 16.40 |
| 5 | 3 | 4.33 | 8.35 | 27.74 |
| | 4 | 3.77 | 7.59 | 26.05 |
| | 5 | 3.26 | 6.80 | 23.91 |
| | 6 | 2.86 | 6.10 | 21.80 |
| 6 | 3 | 5.39 | 9.98 | 30.30 |
| | 4 | 4.86 | 9.37 | 29.62 |
| | 5 | 4.32 | 8.64 | 28.06 |
| | 6 | 3.86 | 7.94 | 26.27 |

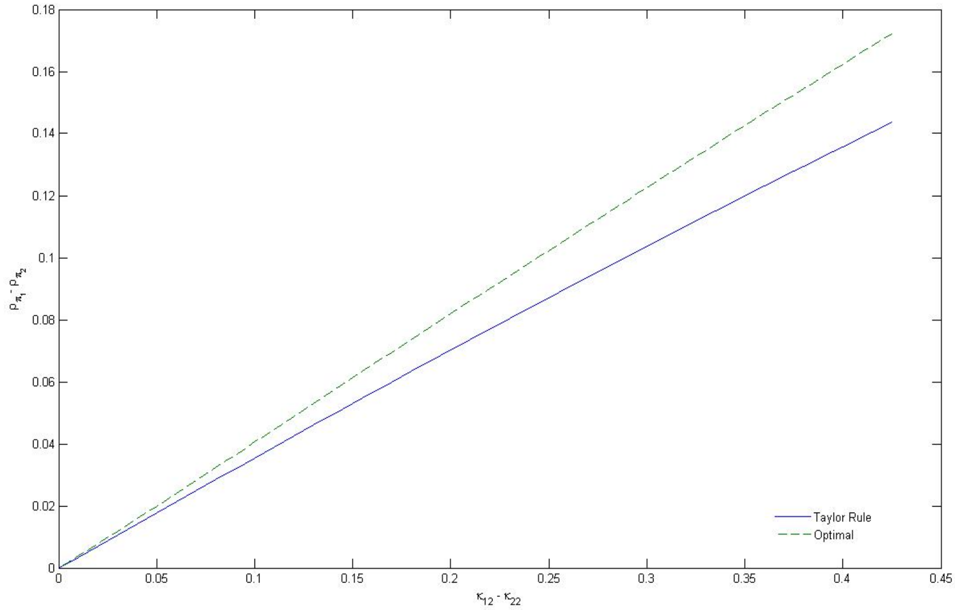
$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The values are $\zeta$ in the optimal inflation targeting rule (22).

Table 3(b): Welfare Cost of Inflation Targeting when Second Sector's Inflation is 0.3 points higher

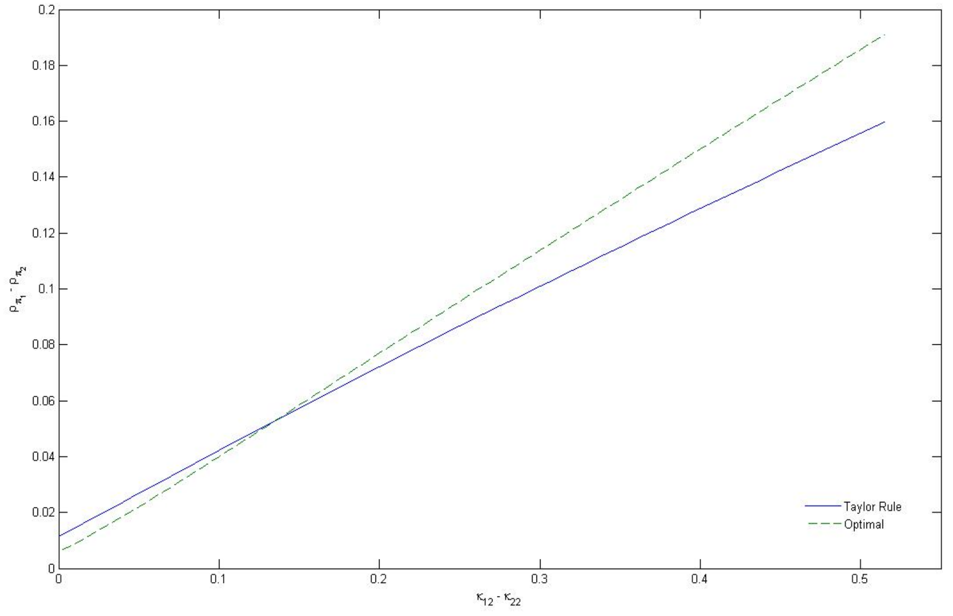
| | | $\psi_2$ | | |
|---|---|---|---|---|
| $D_2$ | $D_1$ | 0.3 | 0.5 | 0.8 |
| 3 | 3 | 11.38 | 21.48 | 51.79 |
| | 4 | 11.97 | 23.23 | 53.47 |
| | 5 | 12.00 | 23.72 | 53.97 |
| | 6 | 11.79 | 23.57 | 53.76 |
| 4 | 3 | | 25.19 | 56.35 |
| | 4 | | 27.95 | 58.21 |
| | 5 | | 29.33 | 58.73 |
| | 6 | | 29.88 | 58.68 |
| 5 | 3 | | 28.41 | 61.60 |
| | 4 | | 32.02 | 63.84 |
| | 5 | | 34.14 | 64.24 |
| | 6 | | 35.24 | 64.03 |
| 6 | 3 | | 31.00 | 69.27 |
| | 4 | | 35.40 | 81.70 |
| | 5 | | 38.19 | 92.54 |
| | 6 | | 39.79 | 69.34 |

$D_2$ is the duration of prices in the second sector and $D_1$ is that of the first sector. $\psi_2$ is the fraction of backward looking price setters in the second sector. The values are $\zeta$ in the optimal inflation targeting rule (22).

Figure 1: Difference in Serial Correlation in Sectoral Inflations

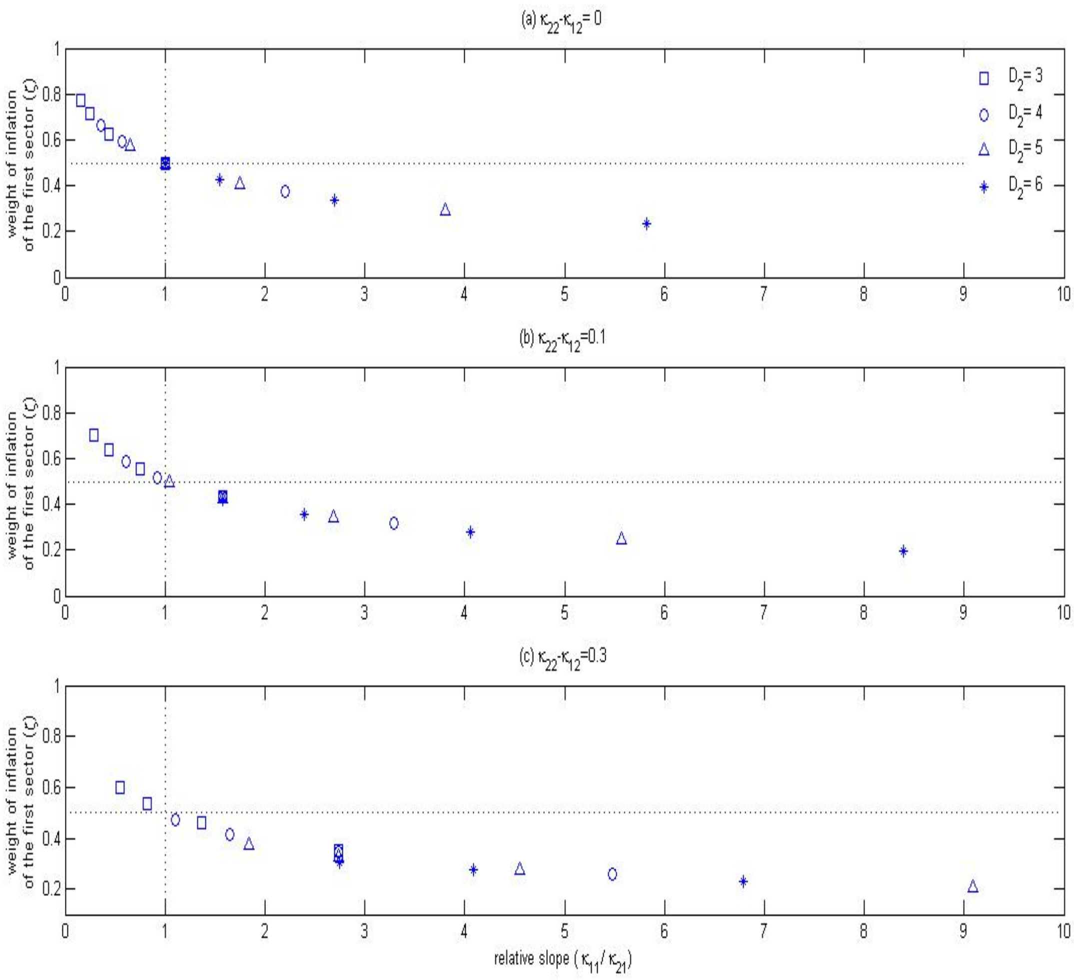



(a)The difference in serial correlations, $\rho_{\pi_i}$, as a function of the difference in the coefficients of the lagged inflation in the sectoral NKPCs, $\kappa_{i2}$. Duration of price is 3 quarters and fraction of backward looking price setters is 0.5 in the second sector and duration of price is 3 quarters in the first sector.



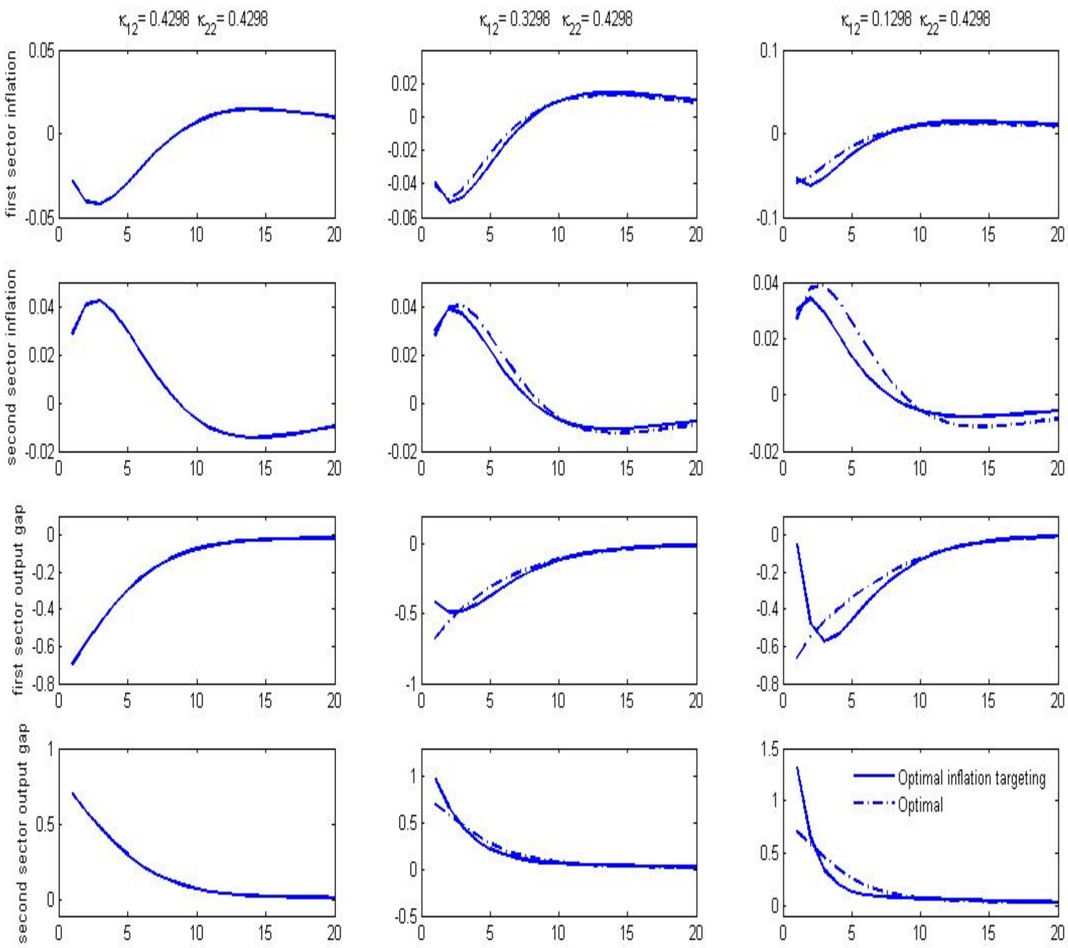

(b)The difference in serial correlations, $\rho_{\pi_i}$, as a function of the difference in the coefficients of the lagged inflation in the sectoral NKPCs, $\kappa_{i2}$. Duration of price is 3 quarters and fraction of backward looking price setters is 0.5 in the second sector and duration of price is 4 quarters in the first sector.

Figure 2: Optimal Weight and Relative Slope of the NKPC



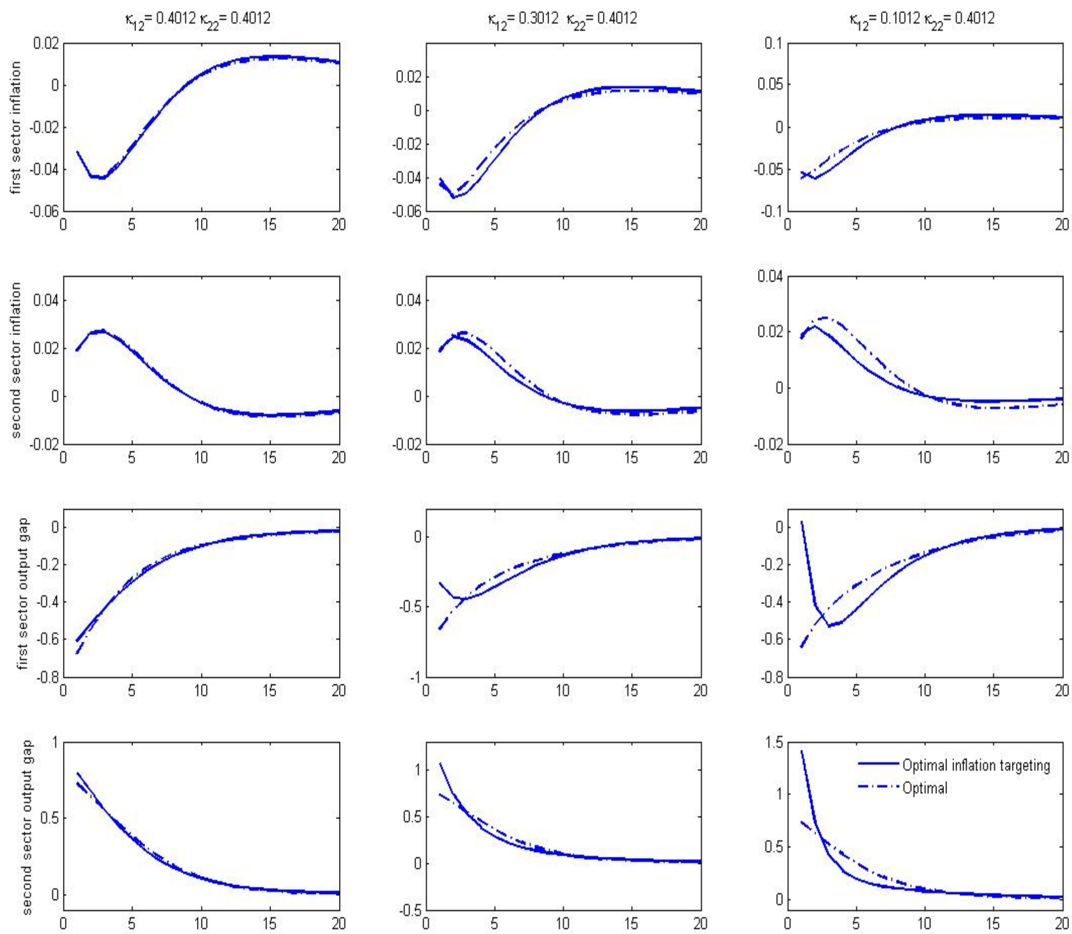

The optimal weight attached to the inflation of the first sector as a function of the relative slope of the sectoral NKPCs for the cases when the differential in degree of inflation persistence is 0, 0.1 and 0.3 points and fraction of backward looking producers is 0.5.

Figure 3: Impulse response functions to a negative supply shock to the second sector



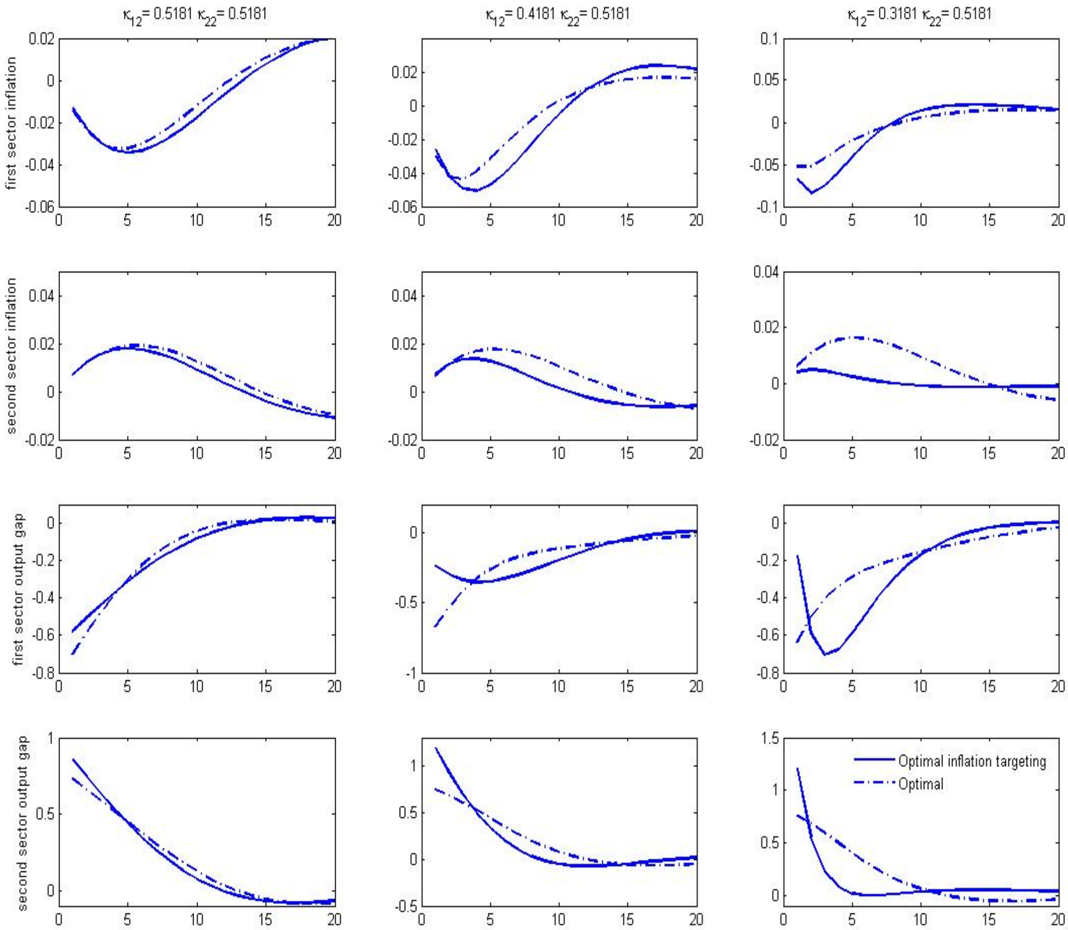

Impulse response functions to a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 3$, $D_1 = 3$ and $\psi_2$= 0.5. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.5, 0.432 and 0.354 and optimal inflation targeting implies 0, 4.34 and 21.48 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.

Figure 4: Impulse response functions to a negative supply shock to the second sector




Impulse response functions to a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 4$, $D_1 = 3$ and $\psi_2 = 0.5$. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.378, 0.319 and 0.261 and optimal inflation targeting implies 0.369, 6.46 and 25.19 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.

Figure 5: Impulse response functions to a negative supply shock to the second sector




Impulse response functions to a negative supply shock to the second sector for the cases that both sectors have same degree of inflation persistence, second sector is more persistent than the first sector by 0.1 points and second sector is more persistent than the first sector by 0.3 points. $D_2 = 4$, $D_1 = 3$ and $\psi_2 = 0.8$. The optimal weights attached to the inflation of the first sector in the inflation targeting rule are 0.343, 0.212 and 0.057 and optimal inflation targeting implies 3.153, 24.70 and 56.35 percent loss when the difference between the degree of inflation persistence across sectors is 0, 0.1 and 0.3 points, respectively.