Lecture 5: Endogenous Margins and the Leverage Cycle

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Leverage ratio and amplification

Leverage ratio: Ratio of assets to net worth.

- Consider the leverage ratio in KM before the shock:

$$L_0^{before} = \frac{\overbrace{q_0 k_1}^{\text{assets}}}{\underbrace{\left(q_t - \frac{q_1}{1+r}\right) k_1}_{\text{net worth}}} = \frac{q_0}{q_0 - \frac{q_1}{1+r}}.$$

- When the economy is near the steady state, $q_0 \sim q_1 \sim q^* = \frac{a}{r}$.
- The leverage ratio, $L_0^{before} \sim \frac{1+r}{r}$. This can be quite large if r is low.
- Leverage ratio can be large in practice. Remember LTCM.
- Leverage ratio of some institutions also seem **procyclical**...

Adrian-Shin (2010): Procyclical leverage for broker-dealers

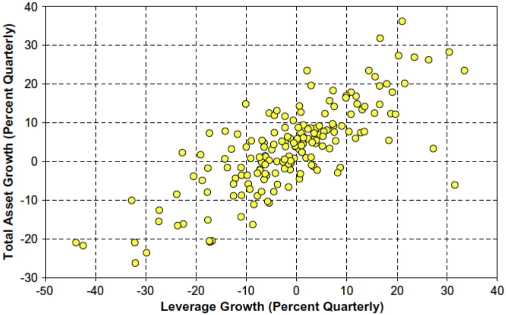


Fig. 4. Total assets and leverage of security brokers and dealers.

- "Net worth" measured as "book equity": Total financial assets minus total liabilities from the US Flow of Funds.

Procyclical leverage would be further destabilizing. Why?

KM model cannot generate procyclical leverage

- Consider the leverage ratio in KM after the shock:

$$L_0(\Delta a) = \frac{q_0(\Delta a)}{q_0(\Delta a) - \frac{q_1(\Delta a)}{1+r}} = \frac{1}{1 - \frac{q_1(\Delta a)}{q_0(\Delta a)} \frac{1}{1+r}}.$$

- Both prices fall, but initial price falls more: $\frac{q_1(\Delta a)}{q_0(\Delta a)} > 1$.
- This would suggest $L_0(\Delta a) > L_0^{before}$. Hard to get procyclicality.

- **Margin** is the inverse of leverage ratio in an asset purchase.

Today: A theory of asset-based leverage, i.e., margins.

- Determination of leverage ratio/margins in this context.
- Procyclical leverage/countercyclical margins. **Leverage cycle.**

Countercyclical margins in the housing market

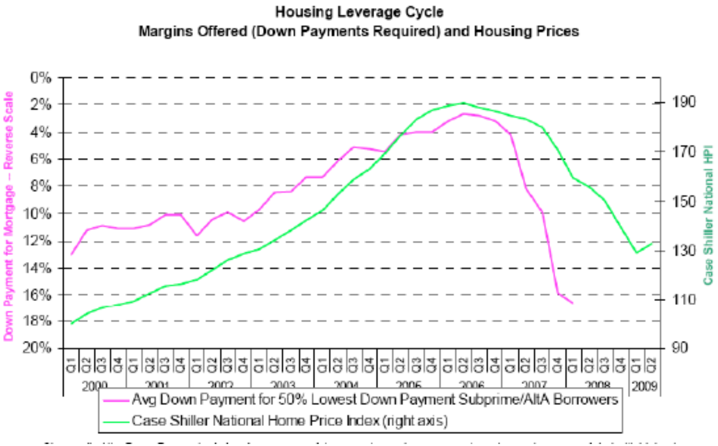
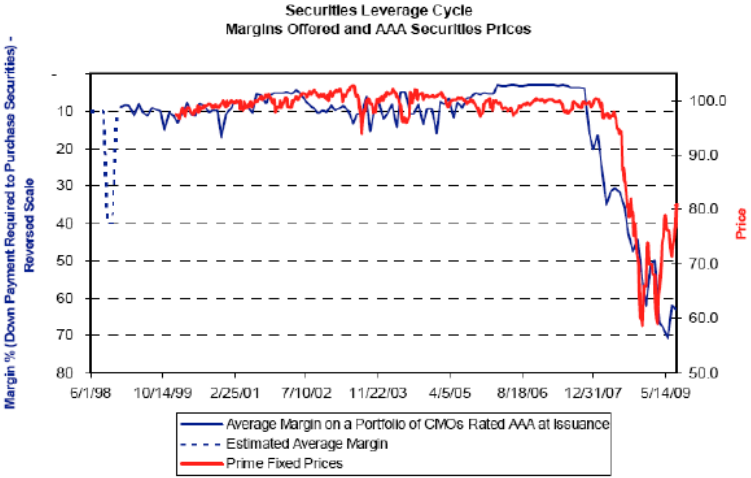


Figure: From Fostel and Geanakoplos (2010).

Countercyclical margins in the MBS market



Note: The chart represents the average margin required by dealers on a hypothetical portfolio of bonds subject to certain adjustments noted below. Observe that the Margin % axis has been reversed, since lower margins are correlated with higher prices.

Basic features of Geanakoplos' leverage models

- **Purely financial assets**: Pay dividends **regardless of the owner**.
  - Different than K-M and much of corporate finance. GE tradition.
- Nonetheless, **heterogeneous valuations** for other reasons.
  - Differences in prefs, beliefs, background risks...
- Heterogeneity generates **demand for borrowing/promises**.
- **All promises are collateralized by assets and non-recourse.**
  - No pledging of endowment other than assets.
  - Default possible and costless. Assets only backed by collateral.
- **Contracts as commodities** in general competitive equilibrium.
  - GE forces "select" traded contracts.

Uncertainty and the leverage cycle

Geanakoplos (2003, 2010) baseline:

- **Only simple debt contracts**.
  - No contingent debt or short selling.
- **Margins (LTVs/riskiness) are endogenously determined**.

Main results:

1. Margins depend on **uncertainty (tail risk)**.
2. Countercyclical margins from changes in uncertainty.

- Start with Simsek (2013) for expositional reasons.
- Then, Geanakoplos (2010) and the leverage cycle.
- Some empirics for bank leverage based on Shin-Adrian et al.

Roadmap

1. Belief disagreements and collateral constraints

2. Leverage cycle

3. Empirics of leverage and the leverage cycle

My paper: Collateral constraint with disagreements

Heterogeneity and collateral: **Endogenous borrowing constraint.**

- Low valuation agents value the collateral less. Reluctant to lend.

Simsek (2013): Understand the constraint for **belief disagreements**.

Main result: Tightness of constraint depends on **type of disagreements**.

Main result: Asymmetric disciplining of optimism

Example: A single risky asset, three future states: G, N, B.

- Pessimists believe each state realized with equal probability.
- **Two types of optimism:**
  1. **Case (D):** Optimists believe probability of B is less than $1/3$.
     $\Longrightarrow$ Margin higher and price closer to pessimists' valuation.
  2. **Case (U):** Optimists believe probability of B is $1/3$. They believe probability of G is more than probability of N.
     $\Longrightarrow$ Margin lower and price closer to optimists' valuation.

Intuition: Asymmetry of debt contract payoffs. Default in bad states.

- Disagreement about downside states $\Longrightarrow$ Tighter constraints.

Basic environment: Belief disagreements about an asset

- One consumption good (a dollar), two dates $\{0, 1\}$.
- Risk neutral traders have resources at date 0, consume at date 1.
- Invest in two ways:
  - Cash: One dollar invested yields one dollar at date 1.
  - **Asset** in fixed supply (of one unit). Trades at price p.
- Asset pays s dollars at date 1, where $s \in \mathcal{S} = \left[s^{\min}, s^{\max}\right]$.
- **Heterogeneous priors**: **Optimists** and **pessimists** with beliefs, F_1, F_0, with:

$$E_1[s] > E_0[s].$$

- **Endowments:** n_1, n_0 dollars at date 0 (asset endowed to outsiders).

Optimists (resp. pessimists) would like to borrow cash (resp. the asset).

Borrowing is subject to a collateral constraint

- A **borrowing contract** is

$$\beta \equiv \left(\underbrace{[\varphi(s)]_{s \in S}}_{\text{promise}}, \underbrace{\alpha}_{\text{asset-collateral}}, \underbrace{\gamma}_{\text{cash-collateral}} \right).$$

- **Collateralized and non-recourse**. Pays:

$$\min(\alpha s + \gamma, \varphi(s)).$$

- **GE treatment:** Traded in anonymous competitive markets at price $q(\beta)$.

Model can account for various borrowing arrangements

Examples of borrowing contracts:

1. **Simple debt contracts:** $\varphi(s) = \varphi$ for some $\varphi \in \mathbb{R}_+$.
2. **Simple short contracts:** $\varphi(s) = \varphi s$ for some $\varphi \in \mathbb{R}_+$.

Next: Baseline with only simple debt contracts:

$$\mathcal{B}^D \equiv \left\{ \left([\varphi(s) \equiv \varphi]_{s \in S}, \ \alpha = 1, \ \gamma = 0 \right) \mid \varphi \in \mathbb{R}_+ \right\}.$$

Denote by **outstanding debt per asset**, φ.

Definition of general equilibrium is standard

Type i traders choose $\left(\mu_i^+, \mu_i^-\right)$ and (a_i, c_i) to maximize their **expected payoffs** subject to:

- **Budget constraint:**

$$pa_i + c_i + \underbrace{\int_{\mathcal{B}^D} q\left(\varphi\right) d\mu_i^+}_{\text{lending}} - \underbrace{\int_{\mathcal{B}^D} q\left(\varphi\right) d\mu_i^-}_{\text{borrowing}} \leq n_i .$$

- **Collateral constraint:** $\mu_i^-\left(\mathcal{B}^D\right) \leq a_i$.

A general equilibrium (GE) is $\left(\hat{p}, q\left(\cdot\right), \left(\hat{a}_i, \hat{c}_i, \hat{\mu}_i^+, \hat{\mu}_i^-\right)_{i\in\{1,0\}}\right)$ s.t. allocations are optimal and markets clear: $\sum_{i\in\{1,0\}} \hat{a}_i = 1$ and $\mu_1^+ + \mu_0^+ = \mu_1^- + \mu_0^-$.

Detour: Consider an alternative principle-agent equilibrium

Alternative to GE: Optimists choose contracts subject to collateral constraint and pessimists' participation constraint.

- When $p < E_1(s)$, optimists invest only in the asset, a_1.
- They choose, φ, which enables them to borrow $a_1 E_0 [\min(s, \varphi)]$.
- Given p, optimists solve:

$$\max_{(a_1,\varphi)\in\mathbb{R}^2_+} \quad a_1 E_1[s] - a_1 E_1[\min(s,\varphi)], \tag{1}$$
$$\text{s.t.} \quad a_1 p = n_1 + a_1 E_0[\min(s,\varphi)].$$

A **principal-agent equilibrium (PAE)** is $(p, (a_1^*, \varphi^*))$, such that optimists' allocation solves problem (1) and the asset market clears.

A regularity condition to capture the notion of optimism

Assumption (A2): The probability distributions F_1 and F_0 satisfy the hazard-rate order ($F_1 \prec_H F_0$), that is:

$$\frac{f_1(s)}{1 - F_1(s)} < \frac{f_0(s)}{1 - F_0(s)} \text{ for each } s \in \left(s^{\min}, s^{\max}\right). \tag{2}$$

- Optimism notion concerns upper-threshold events, $[s, s^{\max}]$.
- Ensures that problem (1) has a unique solution.

Existence, uniqueness, and equivalence of equilibria

Theorem: Under (A1) and (A2):

- There exists a unique PAE, $[p^*, (a_1^*, \varphi^*)]$.
- There exists an essentially unique GE, $\left[(\hat{p}, [q(\cdot)]), (\hat{a}_i, \hat{c}_i, \hat{\mu}_i^+, \hat{\mu}_i^-)_{i \in \{1,0\}}\right]$.
    - The allocations, the asset price, p, and the price of traded debt contracts uniquely determined.
- The PAE and the GE are equivalent, that is:

$$\hat{p} = p^*, \ \hat{a}_1 = a_1^* = 1, \ \hat{\varphi} = \varphi^*, \text{ and } q(\hat{\varphi}) = E_0[\min(s, \varphi^*)].$$

GE allocations are as if optimists have the bargaining power. Intuition?

Optimists' loan choice implies asymmetric disciplining

- Define: **loan riskiness**, $\bar{s} = \varphi$, and **loan size**, $E_0 [\min(s, \bar{s})]$.

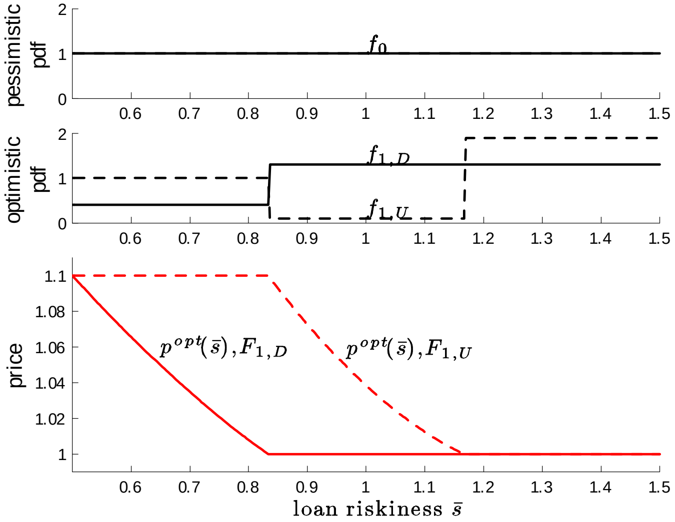
Theorem (Asymmetric Disciplining)

Suppose asset price is given by $p \in (E_0[s], E_1[s])$ and consider optimists' problem (1). The riskiness, $\bar{s}$, of the optimal loan is the unique solution to:

$$\begin{aligned} p &= p^{opt}(\bar{s}) \\ &\equiv F_0(\bar{s}) \int_{s^{\min}}^{\bar{s}} s \frac{dF_0}{F_0(\bar{s})} + (1 - F_0(\bar{s})) \int_{\bar{s}}^{s^{\max}} s \frac{dF_1}{1 - F_1(\bar{s})}. \qquad (3) \end{aligned}$$

- $p^{opt}(\bar{s})$ is like an inverse demand function: Decreasing in $\bar{s}$.
- **Asymmetric disciplining:** Asset is priced with a mixture of beliefs.

Illustration of optimal loan and asymmetric disciplining



Optimists' trade-off: More leverage vs. borrowing costs

- Optimists choose $\bar{s}$ that maximizes the **leveraged return**:

$$\frac{E_1[s] - E_1[\min(s, \bar{s})]}{p - E_0[\min(s, \bar{s})]}.$$

- The condition $p = p^{opt}(\bar{s})$ is the first order condition for this problem.

Optimists' trade-off features two forces:

1. Greater $\bar{s}$ allows to leverage the unleveraged return:

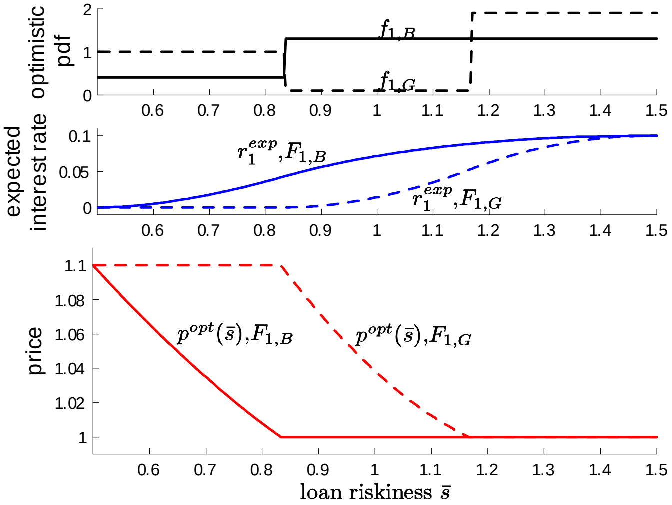
$$R^U \equiv \frac{E_1[s]}{p} > 1.$$

2. Greater $\bar{s}$ is also costlier. Optimists' **perceived interest rate**

$$1 + r_1^{per}(\bar{s}) \equiv \frac{E_1[\min(s, \bar{s})]}{E_0[\min(s, \bar{s})]}$$

is greater than benchmark and strictly increasing in $\bar{s}$.

Intuition for the asymmetric disciplining result



Equilibrium price is determined by asset market clearing

- Optimists' asset demand is:

$$a_1 = \frac{n_1}{p - E_0\left[\min\left(s, \bar{s}\right)\right]}.$$

- **Market clearing:** Set demand equal to supply (1 unit):

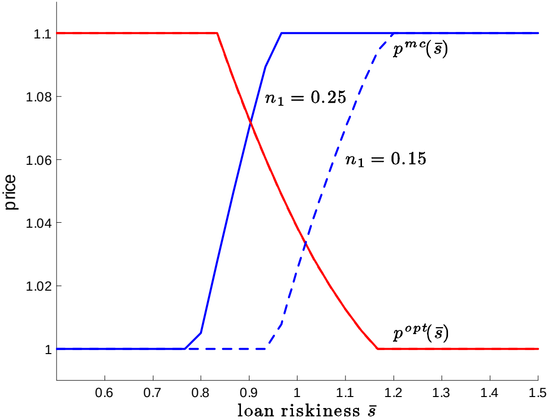
$$p = p^{mc}\left(\bar{s}\right) \equiv n_1 + E_0\left[\min\left(s, \bar{s}\right)\right].$$

  Increasing relation between p and $\bar{s}$.

The equilibrium, $(p, \bar{s}^*)$, is the unique solution to:

$$p = p^{mc}\left(\bar{s}\right) = p^{opt}\left(\bar{s}\right).$$

Illustration of equilibrium



Skewness is formalized by single crossing of hazard rates

- Obtain the comparative statics for $p, \bar{s}^*$ and the margin,

$$m \equiv \frac{p - E_0\left[\min\left(s, \bar{s}^*\right)\right]}{p}.$$

Definition (Upside Skew of Optimism)

Optimism of $\tilde{F}_1$ is *skewed more to upside* than F_1, i.e., $\tilde{F}_1 \succeq_U F_1$, iff:

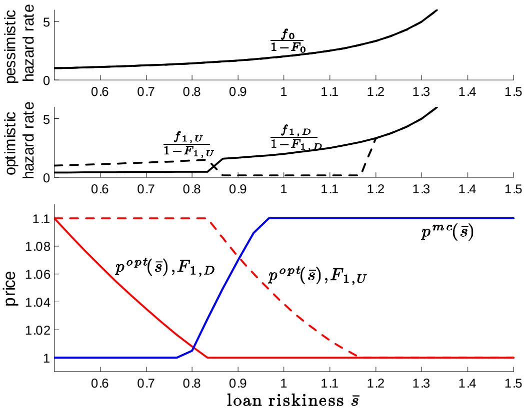
(a) $E\left[s\ ;\tilde{F}_1\right] = E\left[s;\ F_1\right]$.

(b) The hazard rates satisfy the (weak) single crossing condition:

$$\begin{cases} \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s)} \geq \frac{f_1(s)}{1-F_1(s)} \text{ if } s < s^U, \\ \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s)} \leq \frac{f_1(s)}{1-F_1(s)} \text{ if } s > s^U, \end{cases} \text{ for some } s^U \in S.$$

What investors disagree about matters

- **Theorem:** If optimists' prior is changed to $\tilde{F}_1 \succeq_U F_1$, then: the asset price p and the loan riskiness $\bar{s}^*$ weakly increase, and the margin m weakly decreases.



Additional results and taking stock

- Level of disagreement has ambiguous effects.
  - Type of disagreement more important.
- Results are robust to allowing for short selling.
  - Asymmetric disciplining of pessimism. Complementary.
- Richer contracts: Can replicate AD outcomes.
  - Bang-bang contracts as in Innes (1990).
  - Both asset and cash are split. Financial innovation?
- A theory of countercyclical margins: Shifts in type of disagreement.
  - Bad times: Tail risk and downside disagreement.

Next: Geanakoplos' model to formalize and illustrate the leverage cycle.

Roadmap

1. Belief disagreements and collateral constraints
2. Leverage cycle
3. Empirics of leverage and the leverage cycle

Geanakoplos' (2003, 2010) two state model

Geanakoplos baseline: Same setting as before, with two departures:

1. Two continuation states, $s \in \{U, D\}$.
2. Continuum of beliefs. Trader with type $h \in [0, 1]$ believes probability of U is h.

First consider only the first departure. This is the earlier model with $S = [D, U]$ and dF_0 and dF_1 that put all weight on states D and U.

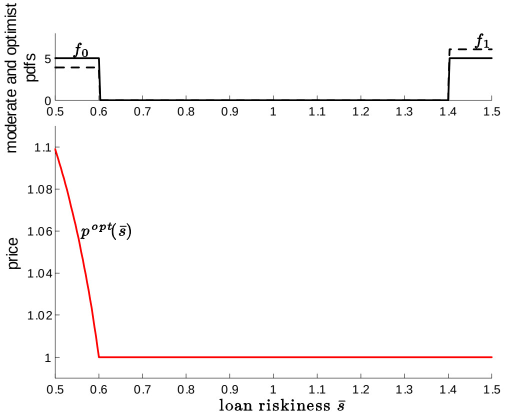
Geanakoplos as a special case of the earlier model

- Debt contract with promise $\varphi \in [D, U]$ priced by pessimists at $h_0\varphi + (1 - h_0) D$.
- Given price $p \in [D, U]$, optimists choose φ that maximizes:

$$\max_{\varphi \in [D,U]} \frac{E_1[s] - (h_1\varphi + (1 - h_1) D)}{p - (h_0\varphi + (1 - h_0) D)}. \tag{4}$$

How does $p^{opt}(\bar{s})$ (and thus, the optimal contract) look in this case?

Geanakoplos as a special case of the earlier model



- For any $p \in (E_0[s], E_1[s])$, the optimal contract has riskiness $\bar{s} = D$.
- With two states, **no default**. Loans are **endogenously** fully secured.

Model with a continuum of belief types

- Next consider **continuum of belief-types**.
- Still two dates, $\{0, 1\}$. We will shortly add a third date.
- Types denoted by, h (beliefs for up state), uniformly distributed over $[0, 1]$.
- Each type starts with (exogenous) net worth, $n > D$.

Benchmark with no leverage: There exists a cutoff $\hat{h}$ such that optimists (with $h > \hat{h}$) invest in the asset, and pessimists (with $h < \hat{h}$) invest in the safe asset...

Benchmark with no leverage

- Indifference condition for the **marginal trader**, $\hat{h}$, leads to an **asset pricing equation**:

$$p = \hat{h}U + \left(1 - \hat{h}\right) D. \tag{5}$$

- Cutoff determined by this equation along with **market clearing**:

$$\underbrace{\frac{n}{p}}_{\text{demand by each optimist}} \left(1 - \hat{h}\right) = 1. \tag{6}$$

- This leads to:

$$p^{noLeverage} = \frac{U}{1 + \frac{U-D}{n}} \text{ and } h^{noLeverage} = \frac{1}{1 + \frac{U}{n-D}}.$$

Equilibrium with leverage

- Suppose optimists can borrow.
- Loans are fully secured (no default theorem). Downpayment D.
- Optimists with $h > \hat{h}$ obtain a **leveraged return** of:

$$R(h) \equiv \frac{hU + (1-h)D - D}{p - D}.$$

- Pessimists with $h < \hat{h}$ obtain a return of 1.
- **Asset pricing** equation unchanged**:** Indifference condition for marginal trader is $R\left(\hat{h}\right) = 1$, which still implies (5).
- **Market clearing** becomes:

$$\underbrace{\frac{n}{p - D}}_{\text{demand by each optimist}} \left(1 - \hat{h}\right) = 1. \quad (7)$$

Compare this with Eq. (6) without leverage.

Equilibrium with leverage

- Solving Eqs. (5) and (7), we obtain:

$$p^{leverage} = \frac{U + D\frac{U-D}{n}}{1 + \frac{U-D}{n}} \text{ and } h^{leverage} = \frac{1}{1 + \frac{U-D}{n}}$$

- Check that $h^{leverage} > h^{noLeverage}$ and $p^{leverage} > p^{noLeverage}$.

- Leverage enables optimists to bid up prices higher. In equilibrium, marginal trader is more optimistic and asset price is higher.
- **This opens the way for instability:** Asset prices are sensitive to leverage and margins (coming up).

Dynamic version to illustrate the leverage cycle

- Suppose there is an additional date, 2. News arrive at date 1.

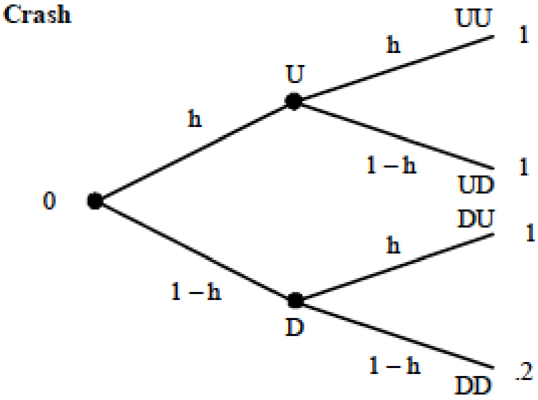
Asset pays only at date 2:

- If there is at least one good news (i.e., UU, UD or DU) asset pays 1.
- If there are two bad news (i.e., DD) asset pays 0.2.

Important ingredient: Bad news and uncertainty go in hand.

- Bad news **creates the possibility of a very bad event**.
- Shift from upside disagreement to downside disagreement.

- Markets open both at dates 0 and date 1. Equilibrium is a collection of asset prices, $(p_0, p_{1,U}, p_{1,D})$, and allocations for type h traders [at both dates 0 and 1] such that traders maximize and markets clear.


Crash
0
h
1 – h
U
D
h
1 – h
h
1 – h
UU
UD
DU
DD
1
1
1
.2

Equilibrium conjecture

Conjecture:

- In period 0, optimists with $h \geq \hat{h}_0$ make a leveraged investment.
- In period $(1, U)$: asset is riskless and sells for $p_{1,U} = U$.
- In period $(1, D)$: optimists from period 0 are wiped out. New optimists, agents in $[\hat{h}_1, \hat{h}_0)$, step in and make a leveraged investment.

Characterization of date 1 equilibrium

- At date $(1, D)$, characterization is identical to the one-period model above, with the only difference that beliefs are distributed over $\left[0, \hat{h}_0\right]$ instead of $[0, 1]$.
- Optimists with $h \in \left[\hat{h}_1, \hat{h}_0\right]$ make a leveraged investment and receive the leveraged return $R_1(h) = \frac{h(1-0.2)}{p_{1,D}-0.2}$.
- Date 1 equilibrium, $\left(p_{1,D}, \hat{h}_1\right)$, characterized by two equations:
  - **Asset pricing**: Indifference condition for marginal trader, $R_1\left(\hat{h}_1\right) = 1$, implies:

$$p_{1,D} = \hat{h}_1 + \left(1 - \hat{h}_1\right) 0.2, \tag{8}$$

  - **Market clearing:**

$$\frac{n}{p_{1,D} - 0.2}\left(\hat{h}_0 - \hat{h}_1\right) = 1. \tag{9}$$

Date 0 equilibrium

Date 0 equilibrium characterization is similar with the following differences:

- Up and down payoffs, U and D, are **endogenous** and are given by $p_{U,1}$ and $p_{D,1}$.
- Marginal trader at date 0 has an option value of saving cash. **Precautionary savings motive**. Intuition? Effect on leverage?

Understanding the precautionary savings motive

- Agent $\hat{h}_0$'s outside option is now:

$$R\left(\hat{h}_0, \text{saving}\right) = \hat{h}_0 + \left(1 - \hat{h}_0\right) \max\left(1, \underbrace{R_1\left(\hat{h}_0\right)}_{\text{this is greater than 1. Why?}}\right).$$

- This is the precautionary savings force. Here, it reduces p_0 and exerts a stabilizing effect.

Characterization of date 0 equilibrium

Date 0 equilibrium, $\left(p_0, \hat{h}_0\right)$, is also characterized by two equations:

- The indifference condition for date 0 marginal trader:

$$\frac{\hat{h}_0 \left(1 - p_{1,D}\right)}{p_0 - p_{1,D}} = \hat{h}_0 + \left(1 - \hat{h}_0\right) \frac{\hat{h}_0 \left(1 - 0.2\right)}{p_{1,D} - 0.2} \quad (10)$$

- Market clearing at date 0:

$$\frac{n}{p_0 - p_{1,D}} \left(1 - \hat{h}_0\right) = 1. \quad (11)$$

- **Equilibrium** $\left(\hat{h}_0, p_{0,D}, \hat{h}_1, p_{1,D}\right)$ is the solution to four equations: (8), (9), (10), (11).
- Solve equilibrium numerically. For $n = 0.68$, should give:

$$p_0 = 0.68, \; p_{1,D} = 0.43, \; \hat{h}_0 = 0.63, \; \hat{h}_1 = 0.29.$$

Main result: Countercyclical margins and leverage cycle

Three factors contribute to the price crash:

1. **Bad news** that lower expected value of asset for all agents.
2. **Net worth channel:** Loss of net worth for most optimistic investors. Asset sold to lower valuation users.
3. **Countercyclical margins** (new destabilizing element that comes from **increased tail risk** and **endogenous margins**).
   - Margin at date 0: $\frac{p_0 - p_{1,D}}{p_0} = \frac{0.68 - 0.43}{0.68} \simeq 22\%$.
   - Margin at date 1: $\frac{p_{1,D} - 0.2}{p_{1,D}} = \frac{0.43 - 0.2}{0.43} \simeq 53\%$.

Leverage cycle: Leverage move together with prices.

Key ingredient: Bad news and uncertainty go hand-in-hand.

Roadmap

1. Belief disagreements and collateral constraints
2. Leverage cycle
3. Empirics of leverage and the leverage cycle

Taking leverage theories to data

- These models emphasize **leverage ratio of Es** for investment/prices.
- Leverage ratio is in turn determined by **tail risk** (extrapolating a bit).
- There is some evidence for these (perhaps for different reasons) when Es are viewed as **banks/broker-dealers.**
- Banks' investment important since it determines credit as in HT.
- Shin, Adrian, and coauthors push this view. **Next:** Brief discussion:
  1. Adrian and Shin (2013): "Procyclical Leverage and Value-at-Risk."
  2. Adrian, Moench, and Shin (2013): "Leverage Asset Pricing."

Measuring leverage ratio for banks/broker-dealers

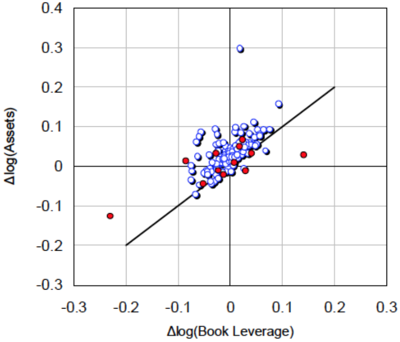
- Challenge: How to measure bank/broker-dealer leverage ratio?
- Two possibilities: **Book leverage** or **market-value leverage**.

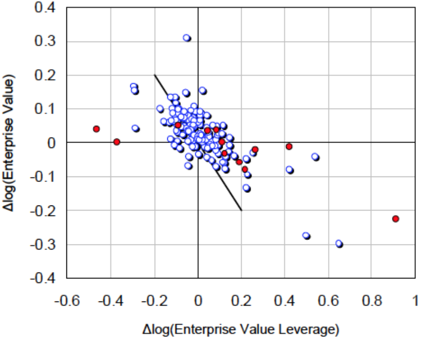
- Define "Book equity" as: Financial assets minus liabilities.
- **Book leverage** is financial assets divided by book equity.

- Define "net worth" as market capitalization.
- Define "enterprise value" as net worth plus debt.
- **Market/enterprise value leverage** is this divided by net worth.

It turns out the two measures behave very differently...

Measuring leverage ratio for banks/broker-dealers





Measuring leverage ratio for banks/broker-dealers

Which definition is **conceptually** more relevant for us?

- Recall we have a theory of asset-based leverage/margins.
- For banks, book equity reflects mostly margins on financial assets.
- In contrast, net worth contains claims to future profits/fees etc.
- Bank equity appears more appropriate in our context.

Book leverage also more relevant **empirically** for **asset pricing**:

- AMS run a horse between two measures. Book leverage wins.

But question is not completely settled. Shin-Krishnamurthy debate.

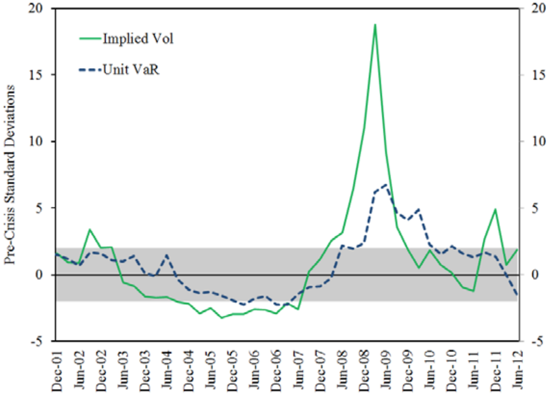
Measuring tail risk for banks/broker-dealers

- Another challenge: How to measure tail risk?
- In practice, banks/regulators use **Value-at-Risk** to assess health:

$$\text{Prob}\,(A < A_0 - V) \leq 1 - c.$$

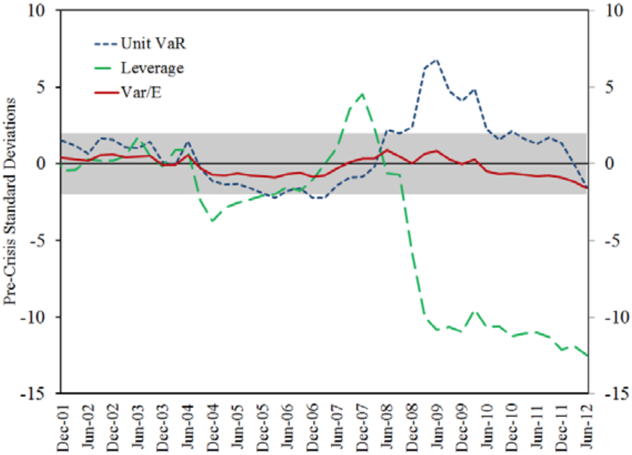
  - Here, A_0 is initial or some benchmark value of assets.
  - A is the end-of-period random value of assets.
  - c is the confidence level. Typically 99% or 95%.
  - V is the **Value-at-Risk** at c over a given horizon.
- Define also **unit VaR** as $v = V/A_0$, VaR per dollar invested.

Banks' VaRs and their implied volatility



- Banks' self-reported VaRs are highly correlated with implied vols.
- Dramatic increase in VaR (extreme losses) during the crisis.

Banks' leverage ratios are correlated with their VaRs



- Consistent with (a broad interpretation of) Geanakoplos (2010).

This suggests a rule of VaR-based leverage

- Interestingly, V/E (VaR divided by book equity) roughly constant.
- Based on this observation, Shin-Adrian propose the rule:

$$E = V, \text{ where recall } \text{Prob}\left(A < A_0 - V\right) \leq 1 - c.$$

- Idea: Banks take E as given. They adjust A_0 **by adjusting their debt** so as to keep V equal to E.
- What happens to A_0 and debt as uncertainty increases/decreases?
- This also give a simple "rule" for leverage ratio:

$$L = \frac{A_0}{E} = \frac{A_0}{V} = \frac{1}{v}, \text{ and thus } \ln L = -\ln v.$$

VaR based leverage holds up in the data

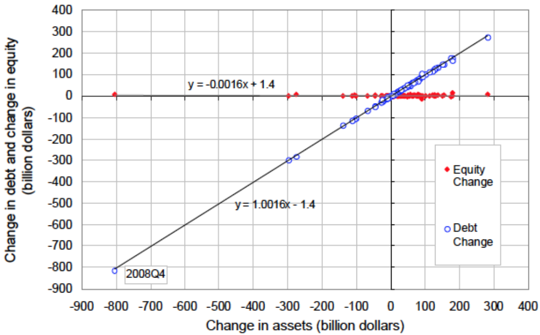


Figure 2: Scatter chart of changes in debt and equity to changes in assets of the US broker dealer sector (1990Q1 - 2012Q2) (Source: Federal Reserve Flow of Funds)

- As predicted, banks seem to adjust assets by changing their debt.
- Interestingly, E seems not only "exogenous" but also fairly sticky.

VaR based leverage holds up in the data

Table 2: **Panel regressions for leverage.** This table reports regressions for the determinant of leverage of the five US broker dealers. The dependent variable is log leverage. Column 1 is the OLS regression for the pooled sample. Columns 2 to 4 are fixed effects panel regressions. 3 and 4 use clustered standard errors at the bank level. Column 5 uses the GEE (generalized estimation equation) method for averaged effects across banks (Hardin and Hilbe (2003)). t statistics are in parantheses in columns 1 to 4. Column 5 reports z scores.

Dependent variable: log leverage (t or z stat in parantheses)	1	2	3	4	5
log unit VaR	-0.479***	-0.384***	-0.384**	-0.421**	-0.426***
	(-11.08)	(-9.2)	(-2.17)	(-2.99)	(-3.12)
implied vol				0.002	0.002
				(0.85)	(0.87)
constant	-1.089	-0.247	-0.247	-0.630	-0.689
	(-2.82)	(-0.66)	(-0.16)	(-0.53)	(-0.59)
R^2	0.40	0.32	0.32	0.34	
obs	185	185	185	185	185
F or χ^2	122.7	84.6	4.71	115.7	337.8
est. method	OLS	FE	FE	FE	GEE
clust. err		N	Y	Y	Y

*** and ** indicate significance at 1% and 5% levels, respectively

- Coefficient not exactly 1 but close. VaR-rule useful starting point.
- Suggests: VaR determines banks' investment, and thus credit to Es.

VaR based leverage holds up in the data

- AMS: Leverage ratio also affects asset prices/predicts asset returns:

	MKT	SPX	BAA
		1975Q1 - 2012Q4	
coeff	-0.070***	-0.065**	-0.025**
	[-2.975]	[-2.542]	[-2.122]
coeff-Stambaugh	-0.070***	-0.064**	-0.025**
	[-2.960]	[-2.527]	[-2.117]
R^2	0.056	0.089	0.029
N obs	151.000	151.000	151.000

- Coef: OLS coefficient on lagged broker-dealer leverage growth.
- Adrian-Etula-Muir: BD-leverage is priced risk factor in cross-section.

Taking stock: Endogenous margins and leverage cycle

Geanakoplos: Theory of countercyclical margins/procyclical leverage.

- Heterogeneity represents **endogenous borrowing constraint**.
- With disagreements, tightness depends on the type of uncertainty.
- **Countercyclical margins** from changes in uncertainty/**tail risk**.

Shin-Adrian and coauthors: Empirical evidence for procyclicality of bank leverage, relation to VaR, and implications for asset prices.