Lecture 2: Investment Dynamics, Uncertainty, and the Precautionary Savings Motive

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Roadmap

1. Introduction
2. Stochastic calculus and optimal control
3. Net worth channel in a dynamic setting
4. Risk management and precautionary savings

Dynamics of financial frictions

Last time:

- Investment with financial frictions in an essentially static setting.
- Dynamics and uncertainty generate new issues:
  - Can firms take precaution against **future constraints**?
  - Can firms save their way out of constraints?

Today: Dynamic implications of financial frictions.

The effect of future borrowing constraints

Dynamics and uncertainty generate precautionary behavior:

- Value function is typically **endogenously concave** in internal funds (Froot, Scharfstein, and Stein, 1993).
- This generalizes the net worth channel to firms that might not be immediately constrained.
- This also generates a rationale for risk management and precautionary savings.

Next: Bolton, Chen, Wang (2011) model to illustrate these results.

- But first brief review of stochastic calculus and optimal control.

Roadmap

Brownian motion and diffusion processes are useful tools for modeling uncertainty

- Brownian motion is a stochastic process $\{x_t\}_{t \in \mathbb{R}_+}$ such that: For any two times $t < s$, $x_s - x_t$ is Normally distributed with mean $\mu(s-t)$ and variance $\sigma^2(s-t)$.
- Represented with (where dZ_t is standard BM):

$$dx_t = \underbrace{\mu}_{\text{drift}} dt + \underbrace{\sigma}_{\text{volatility}} dZ_t$$

- More generally, a **diffusion process** is represented with:

$$dx_t = \mu_t(x_t)\,dt + \sigma_t(x_t)\,dZ_t.$$

  Example: Geometric Brownian motion.
- **Useful objects to model uncertainty because of their tractability:** Functions and products of diffusion processes are also diffusion processes. We can do **stochastic calculus** with them.

Stochastic calculus: Ito's lemma

Consider a diffusion process, x_t. Suppose we are interested in $V(x_t)$.

- **Ito's Lemma**:

$$dV(x_t) = V'(x_t)\,dx_t + \underbrace{\frac{1}{2}V''(x_t)\,\sigma_t^2}_{\text{adjustment to drift}}\,dt$$

Intuition: BM has a lot of volatility even in small intervals, which contributes to the drift **whenever** $V(\cdot)$ **is convex or concave.**

- Example: if $dx_t = \mu dt + \sigma dZ_t$, then

$$dV(x_t) = \left(V'(x_t)\,\mu + \frac{1}{2}V''(x_t)\,\sigma^2\right)dt + V'(x_t)\,\sigma dZ_t.$$

Optimization problem with diffusion processes

- We are typically interested in problems that look like:

$$\begin{aligned} V(x_0) &= \max_{\{y_t \in Y_t\}_{t=0}^{\infty}} E_0\left[\int_0^{\infty} e^{-rt} u_t(x_t, y_t)\, dt\right], \\ \text{s.t. } dx_t &= \mu(x_t, y_t)\, dt + \sigma(x_t, y_t)\, dZ_t. \end{aligned}$$

  - $u_t(x_t, y_t)$ is the flow utility.
  - x_t is the state variable,which follows a diffusion process.
  - y_t is the control, which can also affect the drift or the volatility of x_t.
- In general it is difficult even to calculate the value integral, let alone doing optimization. But suppose problem is stationary (i.e., trade-offs only depend on the state variable x). Then, we can apply the dynamic programming approach.

Dynamic programming approach

Consider instead the **Hamilton-Jacobi-Bellman equation**:

$$\underbrace{rV(x)}_{\text{flow value}} = \max_{y \in Y} u(x,y) + \mu^V(x,y),$$

$$= \max_{y \in Y} \underbrace{u(x,y)}_{\text{flow utility}} + \underbrace{\mu(x,y)V'(x)}_{\text{gains (or losses) from drift}} + \underbrace{\frac{1}{2}\sigma(x,y)^2 V''(x)}_{\text{from volatility (diffusion)}}$$

This approach:

1. characterizes the optimal control,
2. characterizes the value function (as solution to a differential equation),
3. has economic interpretation.

The value function is the solution to a differential equation

- After substituting the optimal control y^* [which may depend on x, $V'(x)$ and $V''(x)$], the HJB eq. reduces to:

$$\rho V(x) = g(x, y^*) + \mu(x, y^*) V'(x) + \frac{1}{2}\sigma(x, y^*)^2 V''(x). \qquad (1)$$

- This is a second order differential equation. Typically has many solutions. The one that we are interested in [corresponding to the value integral] is pinned down by **two boundary conditions**.
- Use the economic problem to identify the boundary conditions.

Dynamic programming can also deal with barriers

Some economic problems might feature **barriers**: Suppose x_t follows a diffusion process inside the range (a, b), but it is not allowed to get outside the range. Examples of actions at barriers:

- **Stopping barrier:** If x_t hits a, then process stops and yields $W(x_t)$.
- **Resetting barrier:** If x_t hits a, then reset at some, x^* at a cost c.

Dynamic programming approach also enables us to:

1. Characterize $V(\cdot)$ when there are **exogenous barriers**,
2. Do optimization over the **endogenous choice of barriers**.

The action at barriers generate value matching conditions

The action at barriers typically imply boundary conditions:

- **Stopping** in the above example implies: $V(a) = W(a)$.
- **Resetting** implies: $V(b) = V(x^*) - c(b, x^*)$, where $c(b, x^*)$ is cost.

Variations of these conditions apply depending on the economic problem.

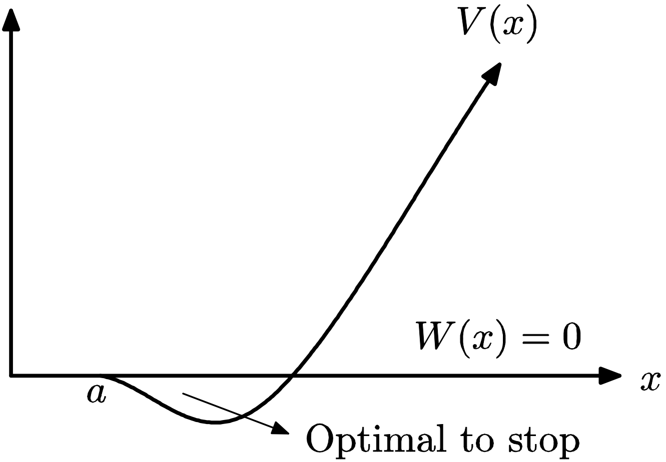
How to do optimization over barriers?

- Suppose the barrier is a choice variable. Then, the problem is to solve the differential eq. along with the endogenous barrier(s).
- Solving for the endogenous barrier requires an additional condition.
- Optimal choice of boundaries gives us this condition.
- **Example:** Choice of stopping barrier, a, implies the **smooth pasting condition**:

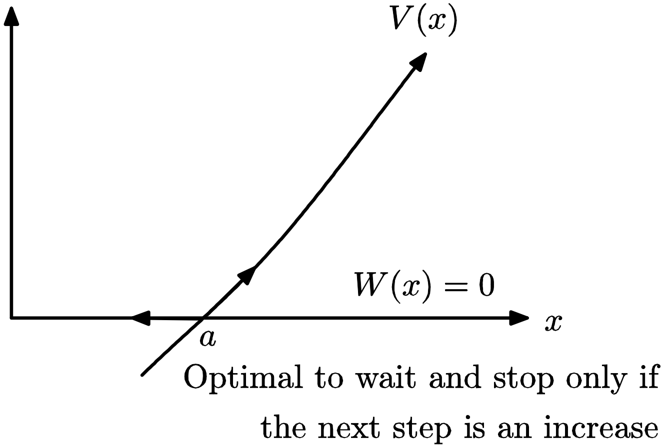
$$V'(a) = W'(a).$$

  Illustrated in the next two figures for the case $W(x) = 0$.
- More generally, one derivative beyond value matching.
- For more on barriers, see Dixit (1993), "The Art of Smooth Pasting."

Why V'(a)<0 cannot be an optimal stopping point



Why V'(a)>0 cannot be an optimal stopping point



Roadmap

A dynamic model of financial frictions and investment

Bolton, Chen, Wang (2011) key features**:**

- Constant investment opportunities.
- Financial constraints:
  - **Equity:** Variable and **fixed** costs.
  - Debt: None. Credit line easy to add.
  - **Cash:** Carry costs. Shortcut to make firms pay dividends.
- Main results:
  - q-Theory with financial frictions.
  - Concave value function (over cash) with rich implications. Framework to analyze jointly firms' risk and cash management decisions.

The basic environment

- Firm with capital K_t over time-increment dt generates revenue:

$$K_t dA_t, \text{ where } dA_t = \mu dt + \sigma dZ_t \text{ captures iid shocks.}$$

- Firm's net profits over time increment dt are given by:

$$dY_t = K_t dA_t - (I_t + G(I_t, K_t))\, dt.$$

- Firm chooses investment. Evolution of capital:

$$dK_t = (I_t - \delta K_t)\, dt, \text{ for } t \geq 0. \tag{2}$$

- Adjustment costs are CRS as in standard q-theory:

$$G(I, K) = g(i)\, K, \text{ where } i = \frac{I}{K} \text{ and } g(i) = \frac{\theta (i - \nu)^2}{2}.$$

- Costless investment ratio allowed to be some ν (as opposed to 0).

The frictionless benchmark

- Suppose no frictions. Wlog, suppose firm chooses finance-as-you-go (no cash kept in the firm).
- Firm maximizes PDV of net profits $E_0\left[\int_0^\infty e^{-rt} dY_t\right]$:

$$
\begin{aligned}
& E_0\left[\int_0^\infty e^{-rt}\left(K_t\mu - \left(I_t + G\left(I_t, K_t\right)\right)dt + K_t\sigma dZ_t\right)\right] \\
= \;& E_0\left[\int_0^\infty e^{-rt}\left(K_t\mu - \left(I_t + G\left(I_t, K_t\right)\right)\right)dt\right].
\end{aligned}
$$

- So the firm's problem can be written as:

$$
\begin{aligned}
P\left(K_0\right) &= \max E_0\left[\int_0^\infty e^{-rt}\left(K_t\mu - \left(I_t + G\left(I_t, K_t\right)\right)\right)dt\right] \\
\text{s.t. } dK_t &= \left(I_t - \delta K_t\right)dt.
\end{aligned}
$$

- This fits into our DP framework...

The frictionless benchmark

- The HJB equation:

$$rP(K) = \max_I \mu K - I - G(I,K) + \underbrace{(I - \delta K) P_K(K)}_{\text{from drift of } K} + \underbrace{0 \times P_{KK}(K)}_{\text{from volatility of } K}.$$

- Observation: $P(K) = qK$ for $q > 0$. The HJB equation becomes:

$$rq = \max_{i=I/K} \mu - i - \frac{\theta (i - \nu)^2}{2} + q(i - \delta).$$

The frictionless benchmark

- FOC for i gives the standard q theory relation:

$$i^{FB} - \nu = \frac{q^{FB} - 1}{\theta}.$$

- Use this to write the HJB equation as:

$$rq^{FB} = \mu - i^{FB} - \frac{\theta}{2}\left(i^{FB} - \nu\right)^2 + q^{FB}\left(i^{FB} - \delta\right).$$

- Two equations, two unknowns. The solution is:

$$i^{FB} = r + \delta - \sqrt{(r + \delta - \nu)^2 - 2\frac{\mu - (r + \delta)}{\theta}}$$

  Question: Why can't we pick the other solution to the quadratic?

- Investment is above ν (the costless benchmark) only if $\mu > r + \delta$.

Lessons from the frictionless benchmark

Without frictions:

1. Marginal q is equal to average q [from linearity of $P(\cdot)$].
2. Temporary shocks (i.e., dZ_t) have no effect on investment.
3. Firm does not need to hedge Z-shocks (even if investors are risk averse). Why?

Consider the case with financial frictions

- Equity financing is allowed: Fixed cost, ϕK, and marginal cost, γ per dollar raised.
    - Crude way to capture constraints. Motivation?
    - Fixed costs are scaled by K to preserve linearity.
- Firm can accumulate cash, W_t.
    - Short term debt not allowed, which means $W_t \geq 0$.

  (Can be relaxed to $W_t \geq -B$, where B is an exogenous debt limit).
- Carry costs: Cash-inside-firm earns $r - \lambda$ as opposed to r.
    - Crude way to capture firms' dividend payments. Motivation?
    - Important for the results. Why? What happens if $\lambda = 0$?
- Other forms of financing (e.g., long term debt) not allowed.

Setting up the firm's problem

- Let U_t^{in} denote (cumulative) external financing, U_t^{out} denote (cumulative) dividend payments, and X_t^{in} denote (cumulative) costs of external financing paid by shareholders.
- Firm's cash position moves according to:

$$dW_t = dY_t + (r - \lambda) W_t dt + \underbrace{dU_t^{in} - dU_t^{out}}_{\text{net cash inflow}},$$

$$\text{where } dY_t = (K_t \mu - I_t - G(I_t, K_t)) dt + K_t \sigma dZ_t$$

- Firm's capital moves according to:

$$dK_t = (I_t - \delta K_t) dt, \text{ for } t \geq 0.$$

- Firm maximizes the following subject to above conditions:

$$\max_{\{I_t, dU_t^{in}, dU_t^{out}\}_{t=0}^{\infty}} P(K_0, W_0) = E_0 \left[\int_0^{\infty} e^{-rt} \left(\underbrace{dU_t^{out} - dU_t^{in} - dX_t^{in}}_{\text{net cash payout}} \right) \right].$$

Approach: Conjecture a solution and show optimality

For a given K, conjecture solution with three regions:

- Internal financing region: $W \in \left[\underline{W}(K), \overline{W}(K)\right]$.
    - In this region, $dU_t^{in} = dU_t^{out} = dX_t^{in} = 0$.
- External financing region: $W \leq \underline{W}(K)$.
    - In this region, reset cash to $W^{reset}(K) \in \left(\underline{W}(K), \overline{W}(K)\right)$.
- Payout region: $W \geq \overline{W}(K)$.
    - In this region, bring W back to $\overline{W}(K)$.

Next: Optimality conditions under this conjecture.

Optimality conditions for the interior region

- In the interior region, the HJB equation holds:

$$rP(K,W) = \max_I \left[\begin{array}{c} (I - \delta K) P_K \\ + [\mu K - I - G(I,K) + (r - \lambda) W] P_W \\ + \frac{1}{2} \sigma^2 K^2 P_{WW} \end{array} \right].$$

Make sure you understand how to derive this expression.

Linearity of the value function simplifies the problem

- Note that value function is linear:

$$P(K,W) = Kp(w)\text{, where } w = \frac{W}{K}.$$

- The thresholds are also linear:

$$\overline{W}(K) = \overline{w}K,\ \underline{W}(K) = \underline{w}K,\text{ and } W^{reset}(K) = w^{reset}K.$$

- **Caveat:** We use $p(w)$ to denote the firm value because this is different than q.
  - Firm's assets: Installed capital (as before) + Cash (new).
  - Thus, average q (average value of capital) is now given by:

$$q(w) = p(w) - w.$$

  - Continue the analysis with firm value, $p(w)$. We will use $q(w)$ for comparisons.

Optimality conditions for the interior region

Under linearity, the HJB equation simplifies to:

$$rp(w) = \max_i \left[\begin{array}{c} (i-\delta)\overbrace{\left(p(w) - p'(w)\,w\right)}^{P_K\text{, i.e., marginal } q} \\ +\overbrace{\left(\mu - i - \frac{\theta(i-\nu)^2}{2} + (r-\lambda)\,w\right)}^{\text{expected net profit}}\overbrace{p'(w)}^{P_W} \\ +\frac{1}{2}\sigma^2\overbrace{p''(w)}^{P_{WW}} \end{array} \right].$$

Optimality of investment gives a version of q theory

FOC for i leads to q-**theory with financial constraints:**

$$i(w) - \nu = \frac{1}{\theta}\left(\frac{\overbrace{p(w) - wp'(w)}^{\text{marginal } q}}{\underbrace{p'(w)}_{\text{marginal value of cash}}} - 1\right). \tag{3}$$

- When cash is abundant, $p'(w) \simeq 1$ (will see) and we have usual q theory.
- In general, $p'(w) > 1$ (will see) and investment will be depressed. Intuition?

How to solve the value function?

- Plugging Eq. (3) into the HJB gives a second order ODE for $p(w)$.
- At the boundaries, the value function must satisfy:

$$\text{Lower boundary:} \quad p(\underline{w}) = p\left(w^{reset}\right) - \phi - (1+\gamma)\left(w^{reset} - \underline{w}\right),$$
$$\text{Upper boundary} \quad : \quad p'(\overline{w}) = 1.$$

- In addition, firm chooses $\overline{w}, \underline{w}$, and w^{reset} optimally:

$$\text{Optimality of lower boundary} \quad : \quad \begin{cases} \text{either } \underline{w} > 0 \text{ and } p'(\underline{w}) = 1+\gamma, \\ \text{or } \underline{w} = 0 \text{ and } p'(0) > 1+\gamma. \end{cases}$$
$$\text{Optimality of upper boundary} \quad : \quad p''(\overline{w}) = 0,$$
$$\text{Optimality of cash reset} \quad : \quad p'\left(w^{reset}\right) = 1+\gamma.$$

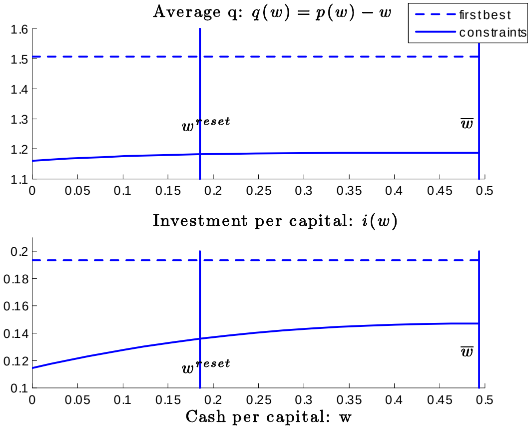
- The parameters $\underline{w}, \overline{w}, w^{reset}$ and the value function $p : [\underline{w}, \overline{w}] \to \mathbb{R}_+$ are found by jointly solving the second order ODE and the five boundary conditions.

Consider a calibration

- Calibration from a more recent version: BCW (2011), "Market timing...".

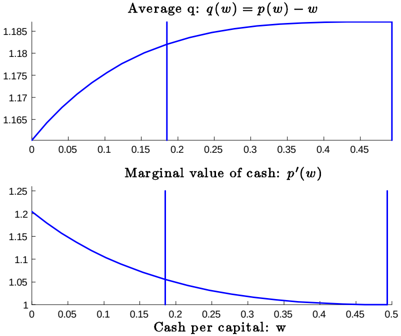
$$
\begin{aligned}
\text{Riskless rate} &: \quad r = 4.34\%, \\
\text{Prod. parameters} &: \quad \mu = 21.2\%,\ \sigma = 20\%, \\
\text{Cash-carry costs} &: \quad \lambda = 1.5\%, \\
\text{Investment parameters} &: \quad \delta = 15\%,\ ,\theta = 6.902, \text{ and } \nu = 12\%, \\
\textbf{Financial costs} &: \quad \underbrace{\phi = 1\%}_{\text{fixed}} \text{ and } \underbrace{\gamma = 6\%}_{\text{marginal}}.
\end{aligned}
$$

Firm value and investment are below the first best



Lower boundary turns out to be $\underline{w} = 0$. Intuition?

Firm value is concave in cash



Equivalently, marginal value of cash is above one and decreasing.

Why is the value function concave?

Value function is concave because of **future constraints**:

- Firms $w = \overline{w}$ are flush with cash and far from being constrained. Thus: $p'(\overline{w}) = 1$.
- For firms with $w = 0$, cash is essential to relax borrowing constraints (alternative is to cut investment or costly issuance): $p'(0) > 1$.
- Firms in between are not yet constrained, but might become constrained in the future. Thus, $p'(w) > 1$ also for these firms and is decreasing in w.

The logic is general: The value function is typically concave in internal funds.

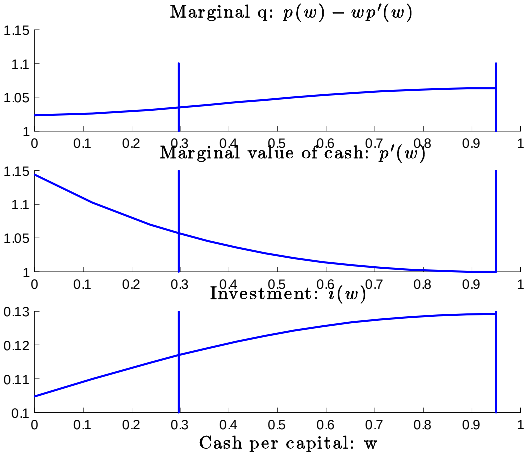
Concavity generalizes the net worth channel

Recall: Investment ~Marginal q / Marginal value of cash.

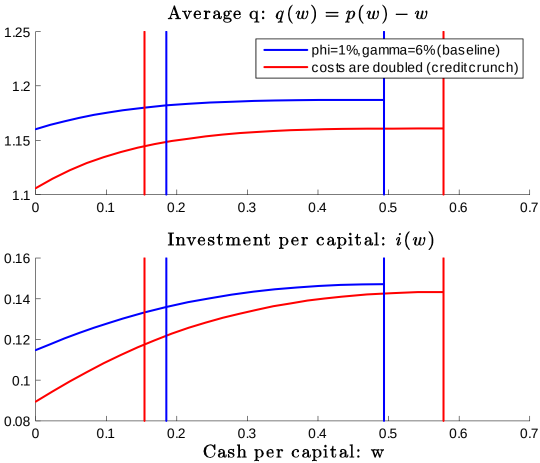
- Marginal value of cash is decreasing in cash.
- Marginal q is increasing in cash (why?).
- Thus, **investment is increasing in cash (or net worth).**

Net worth channel applies even if firms are not yet constrained.

Net worth channel in a dynamic setting

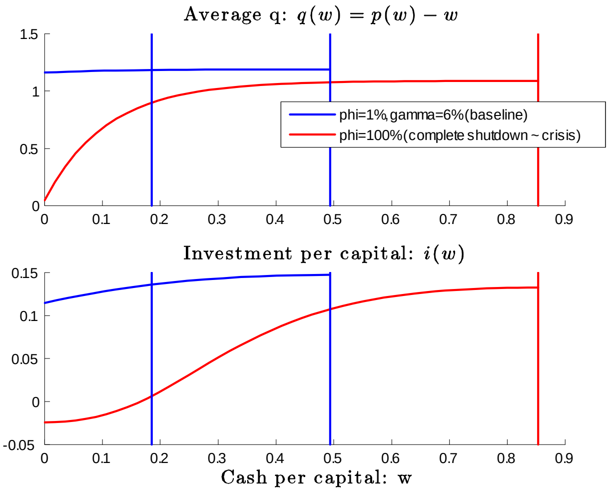


Consider an increase in financing costs (financial crisis)



- Value and investment fall for all firms. More so for firms with low cash.

Consider a severe crisis (complete shutdown of financing)



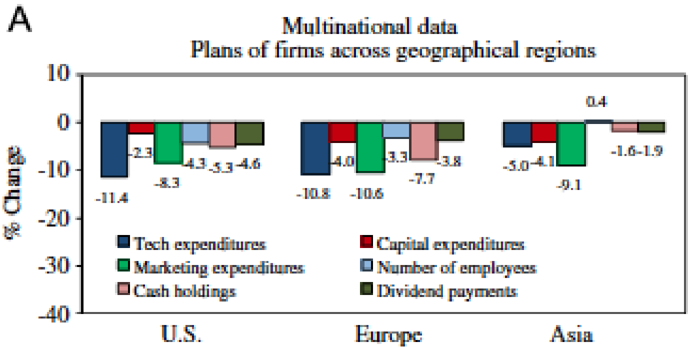
Were firms constrained during the recent crisis?

- Campello, Graham, and Harvey (2010): Survey of 1050 CFOs in 39 countries conducted in Fall 2008: US (574), Europe (192), and Asia (284).
- Questions about financial constraints and future plans.

Results:

- All firms planned cuts in investment—consistent with low Q (driven by very low stock prices) at the time.
- Firms that report to be financially constrained planned much deeper cuts in investment—consistent with the nuanced BC view.

In Fall 2008, firms across the globe planned to reduce various forms of investment (and employment)

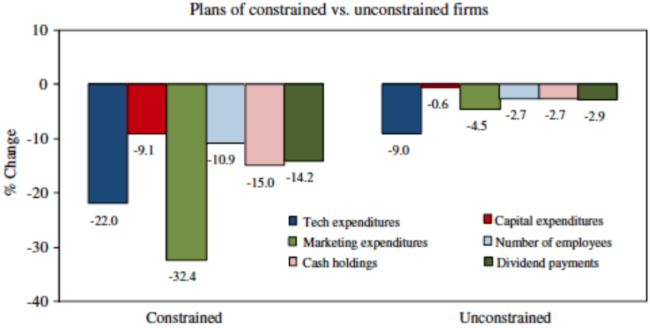


- Figure displays all firms' (not just constrained) change in the policy variable (% per year) as of Fall of 2008.

Firms reported they faced borrowing constraints

- **Borrowing constraints:** Are you affected by difficulties in accessing credit markets?
- In the US sample: 244 indicate **unaffected**, 210 indicate **somewhat affected**, 115 indicate **very affected**.

Main result: Constrained firms in the US planned much larger cuts than unconstrained firms



- They control for many aspects of the firm except for CFOs report of financial constraints and find similar results.

Roadmap

Concavity generates a motive for risk management

- Concavity of value function effectively makes the firm "risk averse."
- This creates a **rationale for risk management (hedging)**.
- Hedging is done **to increase the firm value** (not to make the risk profile more attractive to investors).

Next: Illustrate when the only available asset is market futures.

Firm's problem with risk management

- Futures price (with risk-neutral probabilities) evolves according to:

$$\frac{dF_t}{F_t} = \sigma_m \underbrace{dB_t}_{\text{market risk}} .$$

- Here, B_t, is a standard Brownian motion partially correlated with Z_t, i.e., $E\left[dB_t dZ_t\right] = \rho dt$.
- Suppose there are no costs to hedging (e.g., no margin requirements).
- Let $\psi_t W_t$ denote the firm's futures position at time t.
- The firm's cash flow is now given by: $dW_t =$

$$\left[\begin{array}{c} K_t\left(\mu dt + \sigma dZ_t\right) - \left(I_t + G_t\right) d_t + \left(r - \lambda\right) W_t dt + dU_t^{in} - dU_t^{out} \\ + \underbrace{\psi_t W_t \sigma_m dB_t}_{\text{exposure to market risk}} \end{array}\right].$$

Firm's problem with risk management

Firm's HJB equation (in the interior region) is now given by:

$$rP(K,W) = \max_{I,\psi} \begin{bmatrix} (I-\delta K)P_K \\ +[\mu K - I - G(I,K) + (r-\lambda)W]P_W \\ +\frac{1}{2}\left(\sigma^2 K^2 + \underbrace{2\rho\sigma\sigma_m K\psi W + \sigma_m^2\psi^2 W^2}_{\text{additional terms}}\right)P_{WW} \end{bmatrix}.$$

Optimal risk management

Since $P_{WW} < 0$, ψ is chosen to minimize the additional terms:

$$\psi^* = -\frac{\rho\sigma}{\sigma_m W}.$$

- Firm hedges more when ρ is high, i.e., there are purer hedges.
- If costs are endogenized, firm hedges more when it is cheaper.
- In practice, hedging specific risks is difficult and costly (markets vastly incomplete).

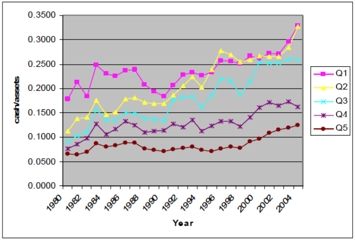
Concavity also generates precautionary savings

- If risks cannot be hedged, then the firm then engages in **precautionary savings**.
- Crude form of hedging. Blanket protection against all kinds of shocks (here Z-shocks).
- The US firms increased their cash holdings in recent years (next slide).
- The real puzzle is why firms, e.g., banks, didn't increase it even more.
- Relatedly: Why pay dividends instead of reducing debt (and increasing equity financing)? See Admati et al. (2010) for a discussion.

Firms in the US secularly increased cash holdings

Figure 1: Average cash ratios by firm size quintile from 1980 to 2004

The sample includes all Compustat firm-year observations from 1980 to 2004 with non-missing data for the book value of total assets and sales revenue for firms incorporated in the U.S. Financial firms (SIC code 6000-6999) and utilities (SIC codes 4900-4999) are also excluded from the sample, yielding a panel of 110,599 observations for 13,237 unique firms. The cash ratio is measured as the ratio of cash and marketable securities to the book value of total assets. Quintiles are sorted on firm size based on the book value of sample firm assets in the fiscal year prior. The first quartile (Q1) is comprised of the smallest firms in the sample while the fifth quartile (Q5) is comprised of the largest firms in the sample.



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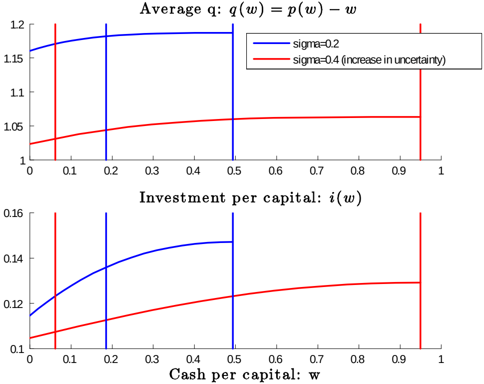
- Kahle, Bates, Stulz (2009): "The average cash ratio increases over the sample period because firms change: their cash flow becomes riskier, they hold fewer inventories and accounts receivable, and are increasingly R&D intensive. The precautionary motive for cash holdings appears to explain the increase in the average cash ratio."

Precautionary savings contributes to amplification

- If there is a sudden increase in uncertainty, it also requires cutting back positive NPV investments and delaying dividend payouts.
- Uncertainty increases during crises for reasons we will come back to.
- Thus, precautionary savings mechanism contributes to amplification.

Another view of a severe crisis: Increase in uncertainty



- Firms accumulate cash by cutting investment and dividend payouts.

Conclusion: Dynamics of financial frictions

BCW (2011): Dynamics of financial frictions. Lessons:

1. **Generalized q theory:** Investment ~Marginal q / Marginal value of cash (or liquid internal funds).
2. With **future constraints**, value function is **endogenously** concave.
   - Generalized net worth channel.
   - Credit crunch reduces investment and dividends for all firms. More so those with low net worth.
   - Firms have a risk management and precautionary savings motive.
   - Uncertainty shocks reduce investment and dividends for all firms. More so those with low net worth.