TRADE AND REAL EXCHANGE RATE: PERMANENT AND TRANSITORY COMPONENTS

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ABSTRACT

JEL Classification:

Keywords:
1. INTRODUCTION

The relationship between exchange rate and trade balance has been an important issue in international economics since the adoption of the floating regime in 1973. After the beginning of floating exchange rate regime, numerous studies have analyzed the impact of currency depreciation on the trade balance. However, these studies find conflicting results.

The early studies that analyze the effect of exchange rate on trade flows focus on the impact of exchange rate volatility on trade flow since the end of fixed exchange rate regime meant an increase of the volatility in the exchange rate markets. In both theoretical and empirical literature, numerous studies analyze this effect but the question of the effect of exchange rate variability on trade is still ambiguous. This point is underlined in a series of overviews of the literature; see McKenzie (1999), Taglioni(2002), Bahmani-Oskooee and Hegerty (2009), Ozturk (2006), Corig and Pugh (2010) and Auboin and Ruta (2013), among others.

After 2000, the focus has shifted towards the relationship between the level of the exchange rate (exchange rate appreciation/depreciation and currency misalignment) and trade flow. Depreciation is typically assumed to improve the trade account in the short run since, by allowing its own currency to lose value relative to those of other countries, a nation's exports which are now cheaper to foreigners increase; while at the same time imports from abroad which are now more expensive in the domestic market reduce. Therefore, the country's balance of trade increases due to the depreciation. There are two immediate effects of currency depreciation; namely the nominal depreciation results in a real depreciation and the rise in relative prices affect the volume of exports and imports. However, empirical findings on these textbook effects suggest that these immediate effects depend on the specific characteristic of the economy.

When we look at the studies analyzing the relationship between exchange rate and trade balance, the twin concepts of the Marshall-Lerner (ML) condition and the J-S Curve phenomenon are compelled. According to the ML condition, the improvement in trade balance due to depreciation depends on whether the sum of import and export demand elasticities exceed unity. There are lots of studies empirically analyzing the ML condition and Bahmani-Oskooee and Niroomand (1998) and Bahmani-Oskooee and Kara (2005) are the most recent studies that provide an estimate of the ML condition using recent advances in time-series econometrics and reveal that the ML condition is not met. In other words, it is found that trade balance continues to deteriorate even if the sum of import and export demand elasticities exceed unity.
Next, since the impact of depreciation on the trade balance is not instantaneous, the studies' focus has shifted to the J-Curve phenomenon, the short run dynamics. The main contributing factors to this phenomenon can be explained by the lag structure in the time of consumers and producers response for the changes in exchange rates. Thus, devaluation creates a short-run deterioration in the trade balance, which will improve in the long run it will improve. Recent review articles by Bahmani-Oskooee and Ratha (2004) and Bahmani-Oskooee and Hegerty (2010) summarizes the main features of these studies and reveal that empirical support for the J-Curve theory is rather weak.

Furthermore, there are recent studies that analyze the impact of currency changes on export growth\(^3\). These studies differ with respect to countries they select. Fang et. al. (2006) analyses the impact of exchange rate depreciation on exports for Asian countries, Bernard and Jensen (2004) study the US, Arslan and van Wijnbergen (1993) focused on Turkish lira depreciation role on exports.

Some recent studies analyze the exchange rate misalignments namely the exchange rate that is above or below the equilibrium exchange rate. To measure the misalignment, studies use different approaches ranging from internal-external balance approach, to the behavior approach and permanent equilibrium approach. From the theoretical concept of the equilibrium exchange rate, some studies (Razin and Collins, 1997; Lee et al., 2008) use the definition of the equilibrium exchange rate. They measure the misalignment as the deviation from equilibrium exchange rate which is the level that both external (asset markets) and internal (productivity hypothesis advanced by Balassa-Samuelson (1964)) markets are balanced in the economy. Moreover, some studies (Rodrik, 2008; Freund and Pierola, 2012; Nicita, 2012) basically regress the real exchange rate on per capita income and the misalignment is simply the difference between the actual and fitted values.

A number of the studies have looked at the empirical relationship between exchange rate misalignment and exports. Some recent studies are Freund and Pierola (2012), Haddad and Pancaro (2010), Nicita (2012). These studies mainly have found that a currency undervaluation has a positive short term impact on exports, but the results are depending on the country characteristics.

\(^3\) See Auboin and Ruta(2013) for a detailed survey of literature.
Moreover, studies that analyze the relationship between exchange rate and growth have also underlined the trade impacts of currency undervaluation (Rodrik, 2008; Di Nino et al., 2011). These studies find that the undervaluation improves growth through expanding the exports.

Recent studies also looked at the exchange rate impact on disaggregated data by analyzing firm’s behavior to a currency appreciation or depreciation. Berman et. al. (2012), Chatterjee et al. (2012), Tang and Zhang (2012) examined how firms react to currency changes for French, China and Brazil respectively. These studies find that large and small firms react differently to the exchange rate changes i.e. the impact of a deprecation on large firms make them increase the mark up, on the other hand small firms change their import prices in case of a currency deprecation. Moreover, large exporters have higher shares in the total exports, thus these firm level studies reflect that the impact of depreciation on total trade flow will be rather weak.

Although there have been numerous papers examining the long run and the short-run relationships between the exchange rate and the trade balance, it is clearly concluded that the empirical evidence has been rather mixed, or inconclusive. This paper on the other hand aims to bridge the gap by looking the components of exchange rate movements and claims that whether exchange rate impact on trade balance depends on the sources of exchange rate movements. In other words, measuring the effects of permanent and temporary movements on exchange rate may become an important issue in determining the relationship between the exchange rate and trade flows.

To decompose exchange rate movements C-F Filter is used and the decomposed permanent and temporary components are derived. After decomposing exchange rate movements, their relationship with trade is analyzed through Gravity Model. By using the Gravity Model of trade, the effect of decomposed exchange rate movements on bilateral exports is examined by adding the permanent and temporary components into Gravity Model as new variables in a panel data analysis. Here the aim of the study is to test the significance of these variables, exante we expect to find a significant impact of the permanent component and an insignificant impact for the temporary component on exports.

The reason behind this expectation can be explained by the definition of these decomposed series. Since temporary component is the cyclical part of the exchange rate and they reflect the transitory changes in the exchange rate, the impact of these movements on real
economy i.e. the international trade flows will be limited. In other words, these temporary movements in the real exchange rate will not be reflected to the trade contracts since these movements are quickly die without affecting the price levels in the long run thus they also will not influence the consumer and producer response to these changes in the exchange rate. However, if the change in the exchange rate corresponds to a change in the trend these movements affect the price level and will probably change the consumer and producer decisions.

To test this hypothesis, first, pooled Ordinary Least Squares (OLS) estimators are utilized to expose the relationship between the components of real exchange rate and bilateral exports. Then, different fixed effects are added into the model to account for the multilateral resistance terms. Silva and Tenreyro (2006) criticize the log-linearization of the empirical model in the presence of heteroskedasticity leading to inconsistent estimates and they show that in the presence of heteroskedasticity the standard methods are biased. They propose a simple Poisson pseudo-maximum-likelihood method (PPML) to overcome this problem. The PPML method provides a robust solution to different patterns of heteroskedasticity and they deal with the zeros that prevail in the trade data. As such, the fixed effects Poisson models with different specifications are estimated to test the relationship between the permanent and the temporary component of the real exchange rate.

Moreover, due to the possible simultaneity bias between bilateral exports, output and also the component of the real exchange rate, we instrument countries income levels and the component of the real exchange rate by their lagged values where we employ different models to solve the endogeneity bias. The inertia in bilateral trade flows that is the countries that trade each other at time t-1 will tend to keep on trading at time t is also considered through Dynamic Gravity Model. The introduction of dynamics into the panel data analysis is modeled by using System Generalized Method of Moments as in Arellano-Bond (1991) and Arellano-Bover (1995) methodologies. Finally, IV-Poisson model that deals with both the zero trade problem and endogenity is estimated to have robust results.

In the empirical analysis, we find that there is no significant relation between temporary components whereas there is a strong and robust negative relationship with the permanent component of the real exchange rate and the bilateral exports. The results indicate that the reason behind the inconclusive results in trade and exchange rate relationship is the mis-measurement of the real exchange rate and if we take out the speculative movements in real exchange rate, the
correct relationship between these variables can be easily identified. Therefore, the effect of a change in real exchange rate on trade volume depends on whether that change reflects a shift in trend or is just a transitory movement.

The plan of this paper is as follows: the first objective is to explain the logical foundations of the gravity model, and then with its theoretical success we model our question through the use of gravity model and explain the data. Then the decomposed components of real exchange rate relationship between bilateral trade flows are analyzed in Section 3 with different estimation techniques. Finally, Section 4 concludes.

2. GRAVITY MODEL

The link between the permanent and the temporary component of real effective exchange rate and the international trade is analyzed by the gravity model. The simply gravity model, which introduced into the field of international economics by Tinbergen (1962), is based on the Physics Gravity Law of Newton. The theory states that bilateral trade flows are positively related to the economic sizes of the two trading countries (measured by their respective GDPs) and negatively to the distance between these countries:

\[
\text{TradeFlow}_{ij} = a\frac{GDP_i \cdot GDP_j}{D_{ij}}
\]

where TradeFlow\(_{ij}\) is the bilateral trade flow from country \(i\) to \(j\), \(GDP_i\) and \(GDP_j\) are the gross domestic products of country \(i\) and \(j\), \(D_{ij}\) is the distance between the two countries and \(a\) is a gravitational constant depending on the units of measurement for mass and force.

The gravity models have now become the standard methodology in empirically studying the bilateral international trade patterns, especially given the increasing emphasis on its strong theoretical basis. Formal theoretical foundations of gravity models have been first provided by Anderson (1979). Bergstrand (1985, 1989) derives the model in reference to monopolistic competition; Deardorff (1998) derives it within a classical Heckscher–Ohlin framework with identical or CES preferences; Eaton and Kortum (2002) develop a Ricardian model of trade in homogenous goods; Haveman and Hummels (2004) have found that gravity model is consistent with incomplete specialization models; and recently Helpman et. al. (2008) develop a theory that predicts positive, as well as zero, trade flows between countries and accounts for firm heterogeneity, trade asymmetries and fixed trade costs. Anderson (2011) and Head and Mayer...
(2013) provide a detailed survey of the theoretical developments underpinning the gravity methodology.

The literature that makes use of the gravity model in empirical studying is surveyed in Anderson and van Wincoop (2004) and Bergstrand and Egger (2011). While many of these studies rely on the simple gravity framework a significant share of these studies further extend the model to include variables such as population (or income per capita), adjacency, common language and colonial links, remoteness, border effects, among others, in the regression analysis. The following analysis is based on such an extended version of the gravity model. In the next sub-section we discuss the data that is used in the analysis, for which the details are provided in Appendix I.

2.1. Data and Measurement

The bilateral trade flows data are obtained from the IMF, Direction of Trade Statistics. The income, population and land data are obtained from WDI, and variables including the distance among countries, the contiguity, common language and common colony are taken from the CEPII Mayer and Zignago (2011) dataset.

The most important independent variable in the following analysis is the decomposed series of real effective exchange rate. The real effective exchange rate is decomposed into permanent and temporary component by C-F Filter. The technical details of the methodology to construct the permanent and temporary component used in this paper are provided in Appendix II.

2.2 Model

The general form of the estimation specification is as follows:

\[ \ln X_{ijt} = \beta_0 + \beta_1 \ln GDP_{it} + \beta_2 \ln GDP_{jt} + \beta_3 \ln Dist_{ijt} + \beta_4 \ln \frac{\text{PER}_{it}}{\text{PER}_{jt}} + \beta_5 \ln \frac{\text{TEMP}_{it}}{\text{TEMP}_{jt}} + \beta_6 \ln Pop_{it} + \beta_7 \ln Pop_{jt} + \beta_8 \ln Land_{it} + \beta_9 \ln Land_{jt} + \beta_{10} \text{Contig}_{ijt} + \beta_{11} \text{ComLang}_{ijt} + \beta_{12} \text{ComCol}_{ijt} + u_{ijt} \]  

(1)

\footnote{Anderson and van Wincoop (2003) show that bilateral trade is determined by relative trade costs and it is crucial to control these multilateral resistance terms in estimating gravity model. Therefore, following the literature, given the difficulty of obtaining the relevant price indices that would allow calculating such trade resistances, I include different cases of fixed effects. This inclusion is standard procedure in the literature; Feenstra (2004) and Baldwin and Taglioni (2006) have shown that including such fixed effects provides similar results to those of Anderson and van Wincoop (2003). The inclusion of these fixed effects results in the dropping some variables from equation 1.}
where $i$ denotes the exporting country and $j$ denotes the importing country, $X_{ijt}$ measures the total exports of country $i$ to country $j$ in millions of US dollars, GDP measures the respective country’s Gross Domestic Product in millions of US dollars, $Dist_{ijt}$ measures the distance between country $i$ and $j$ in nautical miles, $\frac{PER_{it}}{PER_{jt}}$ is the ratio of the permanent component of real effective exchange rate and $\frac{TEMP_{it}}{TEMP_{jt}}$ is the temporary component of the real effective exchange rate that is decomposed by C-F Filter. The ratio of these components of real effective exchange rate that measures the bilateral value of the currency in exporting country $i$ relative to the importing country $j$ is added into the model. $Pop$ is the population, $Land$ stands for land area, $Contig_{ijt}$ is the dummy that takes the value 1 if exporter $i$ and importer $j$ are contiguous and zero otherwise, $ComLang_{ijt}$ is the dummy with value 1 if exporter $i$ and importer $j$ share a common language and zero otherwise, and finally $ComCol_{ijt}$ is the dummy with value 1 if both have had a common colonizer after 1945 and zero otherwise.\(^5\) $u_{ijt}$ is the log-normally distributed disturbance term.

Based on the gravity model we expect to find the coefficient of the exporting and importing countries’ income to be positive and that of the distance between these two countries measure to be negative. The sign for the permanent and temporary component will be negative as we expect a depreciation of a currency increases the exports of that country. These variables are used to test the hypothesis whether exchange rate movements as movements driven by the fundamentals and movements driven by the unobservables have an effect on trade. Here, our claim is movements that are specific to fundamentals may have a significant effect on bilateral trade volumes, whereas speculative movements, which have a transitory effect on the exchange rates, may have an insignificant effect on bilateral trade volumes.

The population variable is expected to represent the country’s potential supply and demand for exports and imports respectively. A country with a large population can much easily specialize in a wide range of commodities and, consequently, may be less dependent on foreign trade leading to a negative coefficient. Alternatively, if the demand factors are dominant the variable might result in a positive effect on exports.

Land variables are assumed to have negative influence on trade. The larger a countries' total area, smaller the fraction of its economic activity that is expected to cross borders and higher

\(^5\) See Mayer and Zignago (2011) for further details.
the probability of it becoming a relatively closed economy. Finally, three dummy variables that shed light on the circumstance of being a neighbor, sharing a common language or having been colonized by a common country are respectively included in the model. The coefficients of all three dummy variables are expected to be positive as their existence will increase the level of bilateral trade.

Table 1 summarizes the expected signs of the coefficients in light of alternative theories.

3. Empirical Results

3.1 Static Panel Gravity Equation

First of all, in testing for the presence of permanent and temporary components of real exchange rate effect on bilateral trade, equation (4.1) is estimated by OLS in different model specifications. The results are reported in Table 2 where the dependent variable which is the total exports of country $i$ to $j$ is estimated for 1994M01-2012M12 period. Firstly, pooled Gravity Model with the components of real exchange rate is reported in column (1) of the Table 2. As theoretically expected, income of these two countries and the distance between them show statistically significant and theoretically expected signs. The coefficients of the population of both countries are found to be positive and significant. Land variables have significant coefficients with expected negative signs. All three factors capturing the contiguity of the two countries, as well as common cultural features such as a shared history or shared language, are found to positively and significantly explain bilateral export patterns.

The main question of interest in this analysis is to test the link between the bilateral trade and the permanent - temporary components of the exchange rate. With this first specification, it is found that there is not a statistically significant relationship between neither the permanent nor the temporary components of the real effective exchange rate and the bilateral exports. The model is improved through the addition of time dummies in the second column but the results change only slightly. The sign of the coefficient of the temporary component becomes negative but it has still insignificant impact.

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6. The Variance Inflation Factor (VIF) scores, which test multicollinearity, are lower than 10 (with an average of 2.09) supporting that there is no multicollinearity problem.

7. Including time fixed effect, we can prevent the bronze medal mistake that Baldwin and Taglioni (2006) define as the inappropriate deflation of trade flows.
Up to now, as Anderson and van Wincoop (2003) found all the coefficients are biased due to the omission of the multilateral resistance terms but we take this into account in the following columns through considering different country fixed effects (Feenstra, 2004 and Baldwin and Taglioni, 2006). In the third column, the model with both exporter and importer fixed effects in addition to time fixed effect is reported. With the inclusion of these fixed effects the coefficient of the permanent component becomes significant. On the other hand, the impact of the temporary component is still found to have an insignificant impact. In the fourth column, including pair dummies along with the time dummies does not change the result of the previous model.

Finally, in the last column time varying exporter and importer fixed effects\(^8\) with pair fixed effect are included into the model. With this model specification, all of the variables are dropped from the model since they are perfectly collinear with the included fixed effects. To determine the impact of the permanent and temporary component of the real effective exchange rate on trade flows we propose two new variables that are driven by the multiplication of the permanent component and the temporary component with the distance. With this parameter, we can generate a time varying dyadic variable that can be estimated with the inclusion of the time varying exporter and importer fixed effects along with the dyadic fixed effects. Here, we claim that the impact of currency movements on bilateral trade flows will be different in terms of the distance between the trading countries. In other words, the impact of currency depreciation will be much more on the exports to a country near the exporting country than a country which is remote from the exporting country. With this claim, we expect a negative sign for the coefficients and as we expect both permanent and the temporary component interacted with the distance are found to have a negative impact on exports. Moreover, the permanent component interacted with the distance has a significant impact on the exports.

While all results remain the same with the inclusion of different types of fixed effects, which possibly include trade resistances alongside many country- and pair-specific factors, it is seen that the permanent component of the real effective exchange rate has an influential factor in the country’s export performance, however the temporary component does not affect the export performance. In short, these results show that appreciation in a country’s currency that is driven by the fundamentals negatively contributes to that country’s exports. However, if the appreciation is a result of a speculative currency trade, it does not affect the country’s export performance.

\(^8\) In a panel setting, the theoretical specification maintains the inclusion of time varying exporter and importer fixed effects.
3.2 Zero Trade

Silva and Tenreyro (2006) criticize the log-linearization of the gravity model in the presence of heteroskedasticity that leads to inconsistent estimates and they show that in the presence of heteroskedasticity the standard methods can severely bias the estimated coefficients. They propose a simple Poisson pseudo-maximum-likelihood method (PPML) to overcome this problem. The PPML method provides not only a robust solution to different patterns of heteroskedasticity but also a natural way to deal with the zeros that prevail in the trade data. In Table 3, models that consider the zero trade problems with different specifications are estimated.

Like in the previous table, as a preliminary step, we present the results with the estimation of the PPML and PPML with time dummies in the first and second column of the table respectively. Different from the previous table, the permanent component has a significant impact on the international trade. On the other hand, the temporary component is statistically insignificant. All other variables are found to be statistically significant.

To consider multilateral resistance terms, exporter and importer fixed effects with the time dummies are considered in Column (3) and paired fixed effects with time dummies are considered in Column (4). The significance of the permanent components remains same in these two models. However, different from the previous results, once the paired fixed effects are included into the model, the temporary component becomes a significant positive impact on bilateral exports. This result can be explained by the J-curve that the effect of a transitory change in exchange rate on the trade flow initially creates a short-run deterioration in the exports. Moreover the unobserved reasons behind the exchange rate depreciation may create uncertainty in the economy that affects the export performance negatively.

Instead of the Poisson specification, in the fifth column negative binomial model with paired and time fixed effects is presented. Although it is known that other count data models like negative binomial are not adequate to estimate gravity model, i.e. negative binomial regression models are not invariant to the scale of the dependent variable, we just report this model as a robustness check and it is seen that our result are still valid even if we consider the negative binomial regression models.

Finally, in the last column we report another specification proposed by Helpman et. al. (2008) taking into account the selection bias and firm heterogeneity. Only the second step estimation results are reported. In the first step, a panel probit model with importer and exporter
fixed effects along with time dummies is estimated. From this step we obtain the predictions of firm heterogeneity (ZHAT) and selection bias (INVMILLS) that we use in the second step estimation that includes importer and exporter fixed effects. The important result of this model confirms the Helpman et. al (2008) results that firm heterogeneity (ZHAT) and selection bias (INVMILLS) have a significant impact on the export performance. The coefficient of the permanent component has also a significant negative impact on export. Different from previous findings, here temporary component is also found to have a significant negative impact on trade flows. An important drawback of this model is that through the estimation of the nonlinear system, time fixed effects are not incorporated into the second step estimations, thus this will be the reason why the temporary component is found to have a negative significant impact.

3.3. Endogeneity

Since international trade flows form a significant part of GDP, which is one of the regressor of the equation, there may be a causality problem in the estimation of the gravity model leading to the previous model’s coefficients to be inconsistent. Moreover, it is also possible that international trade flows and these components of the exchange rate have reverse causality. To control for such problems, in columns (1) and (4) of Table 4, we use the instrumental variable technique with different specifications in estimating the gravity equation. We select lagged GDP variables and lagged permanent and temporary components as possible instruments, according to Hansen’s J-statistics. The first-stage regressions F-statistics are quite high, signaling that the instruments are highly correlated with the independent variable GDP and these components, and that they support the validity of instrument choices.

In the first column, OLS results of the IV estimation with paired and time fixed effects are reported. The main results of the analysis also prevail when overcoming the endogeneity biases, where a country’s exports are negatively associated with the permanent component of the exchange rate however the temporary component has not a significant impact on the bilateral trade between these two countries. In the second column, GMM results for the two step IV estimation having similar results with the OLS are presented.

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9 This model proposed by Helpman, et. al (2008) is applied on a crossection and confirms the importance of firm heterogenity and selection in analyzing the international trade. To take into account these features we apply their framework on a panel setting. However, adapting the approach in Helpman et.al. (2008) to a panel setup is not obvious. Here, we follow Whitten(2012) study to estimate the model.
Moreover, the inertia in bilateral trade flows that is the countries that trade each other at time \( t-1 \) will tend to keep on trading at time \( t \) is considered through Dynamic Gravity Model in the third column of the table. Since the regressors are not strictly exogenous that are correlated with past and possibly current realizations of the disturbance term, Ordinary Least Squares (OLS) estimations would create biased estimates (Bond, 2002). Therefore, the introduction of dynamics into the panel data analysis is modeled by using System Generalized Method of Moments (SGMM) as in Arellano-Bover (1995) and Blundell-Bond (1998) methodologies.

In practice, very remote lags are unlikely to be informative instruments and to check the validity of instruments; we use the Hansen's J test of over identifying restrictions\(^{10}\). We estimate system GMM including country and time dummies and in all GMM estimations, two-step procedure is applied.\(^{11}\) We estimate the gravity equations with GMM using optimal weighting matrix. This optimal weighting matrix makes two-step GMM asymptotically efficient. In order to verify that the error term is not serially correlated, \( m_1 \) and \( m_2 \) statistics are included as tests for first and second order serial correlation.

In the third column, parameter estimates for the Dynamic Gravity equation are presented. Before concentrating on the economic implications of the estimation results two specification tests should be checked in order to make sure GMM results consistency. The first one is that the idiosyncratic error of the estimators be serially uncorrelated. In other words, the null of no autocorrelation at order one should be rejected but for higher orders of the residuals it should not be rejected. The test for the System GMM estimator is presented as statistics \( m_1 \) and \( m_2 \) in Table 4, showing the required results. Moreover, it should be noted the Hansen's J statistics should accept validity of the instrument set. In this System GMM estimation, the instruments for the level equations are specified in addition to the instruments for the first differenced equations.

The results of this model is parallel to our previous results that can be summarized as follows: the permanent component of the real exchange rate has a significant negative effect on this country's export on the other hand, the impact of temporary component of the real exchange rate on the bilateral exports is statistically insignificant.

Finally, in the last column both endogeneity bias and the zero trade problems are taken into account with the estimation of the Gravity Model through IV-Poisson regression model. Overall,

\(^{10}\) Different from Sargan, Hansen tests are robust to heteroscedasticity, albeit they are vulnerable to instrument proliferation (Roodman, 2006, 2008).
\(^{11}\) For comparison, one step procedure is also applied. The results give consistent results, thus they are not tabulated in the study.
regardless of the estimation technique, the permanent component of the real exchange rate is shown to contribute negatively to the extent of bilateral trade between two countries. In other words, all these regressions results point to a very robust negative relationship among the permanent component and the exports. On the other hand, the impact of temporary component of the real exchange rate on the bilateral exports is statistically insignificant. Therefore, it can be concluded that the effect of a change in real exchange rate on trade volume depends on whether that change is driven by a shift in fundamentals or is just driven by a transitory movement.

4. Conclusion

When we look at the literature that analyzes the relationship between exchange rate and trade flow, it is seen that there exist a gap to uncover a link between the exchange rate and trade flow. However, after the beginning of floating exchange rate regime, numerous studies have analyzed the impact of currency depreciation on the trade balance and in the literature of exchange rate and trade relationship, these studies find conflicting results.

Thus, following the first chapter, C-F filter decomposed exchange rate movements are added into Gravity Model and it is found that the permanent component of real exchange rate is effective on trade. On the other hand, the temporary component of the real exchange rate is found to be insignificant. Therefore, it can be concluded that the effect of a change in real exchange rate on trade volume depends on the sources of this change and identifying these sources of variation in exchange rate is important in determining this relationship.
REFERENCES


TABLES

Table 1: Expected Signs of Coefficients

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expected sign</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income of exporting country $i$</td>
<td>$+$</td>
<td>Increased mass, à la gravity model.</td>
</tr>
<tr>
<td>Income of importing country $j$</td>
<td>$+$</td>
<td>Increased mass, à la gravity model.</td>
</tr>
<tr>
<td>Distance between $i$ and $j$</td>
<td>$-$</td>
<td>Increased cost-reducing trade, à la gravity model.</td>
</tr>
<tr>
<td>Permanent component of real effective exchange rate of exporting country $i$ relative to importing country $j$</td>
<td>$-$</td>
<td>A depreciation of a currency increases the exports of that country</td>
</tr>
<tr>
<td>Temporary component of real effective exchange rate of exporting country $i$ relative to importing country $j$</td>
<td>$-$</td>
<td>A depreciation of a currency increases the exports of that country</td>
</tr>
<tr>
<td>Land of exporting country $i$</td>
<td>$-$</td>
<td>The larger a country’s total area, the smaller the fraction of its economic activity that is expected to cross borders and the higher probability of a relatively closed economy.</td>
</tr>
<tr>
<td>Land of importing country $j$</td>
<td>$-$</td>
<td>The larger a country’s total area, the smaller the fraction of its economic activity that is expected to cross borders and the higher probability of a relatively closed economy.</td>
</tr>
<tr>
<td>Population of exporting country $i$</td>
<td>$+/-$</td>
<td>Population is a good approximation for the effects of economies of scale. A country with a large population can more easily specialize in a wide range of commodities and, consequently, may be less dependent on foreign trade, which may lead to a negative coefficient. Alternatively, if the demand factors are dominant, the variable might result in a positive effect on exports.</td>
</tr>
<tr>
<td>Population of importing country $j$</td>
<td>$+/-$</td>
<td>Population is a good approximation for the effects of economies of scale. A country with a large population can more easily specialize in a wide range of commodities and, consequently, may be less dependent on foreign trade, which may lead to a negative coefficient. Alternatively, if the demand factors are dominant, the variable might result in a positive effect on exports.</td>
</tr>
<tr>
<td>Common language</td>
<td>$+$</td>
<td>Its existence will increase the level of bilateral trade.</td>
</tr>
<tr>
<td>Contiguity</td>
<td>$+$</td>
<td>Its existence will increase the level of bilateral trade.</td>
</tr>
<tr>
<td>Common colonization</td>
<td>$+$</td>
<td>Its existence will increase the level of bilateral trade.</td>
</tr>
</tbody>
</table>

Notes: The ex ante expectations of the sign of the coefficients are based on the detailed literature survey conducted by the authors.
Table 2: Static Panel Results

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
<tr>
<td>ln(GDP)_i</td>
<td>ln(GDP)_j</td>
<td>ln(GDP)_i</td>
<td>ln(GDP)_j</td>
<td>ln(GDP)_i</td>
<td>ln(GDP)_j</td>
</tr>
<tr>
<td>ln(Dist)_i</td>
<td>ln(Dist)_j</td>
<td>ln(Dist)_i</td>
<td>ln(Dist)_j</td>
<td>ln(Dist)_i</td>
<td>ln(Dist)_j</td>
</tr>
<tr>
<td>ln(TMP/PER)_i</td>
<td>ln(TMP/PER)_j</td>
<td>ln(TMP/PER)_i</td>
<td>ln(TMP/PER)_j</td>
<td>ln(TMP/PER)_i</td>
<td>ln(TMP/PER)_j</td>
</tr>
<tr>
<td>ln(Pop)_i</td>
<td>ln(Pop)_j</td>
<td>ln(Pop)_i</td>
<td>ln(Pop)_j</td>
<td>ln(Pop)_i</td>
<td>ln(Pop)_j</td>
</tr>
<tr>
<td>Contig_i</td>
<td>Contig_j</td>
<td>Contig_i</td>
<td>Contig_j</td>
<td>Contig_i</td>
<td>Contig_j</td>
</tr>
<tr>
<td>ComCol_i</td>
<td>ComCol_j</td>
<td>ComCol_i</td>
<td>ComCol_j</td>
<td>ComCol_i</td>
<td>ComCol_j</td>
</tr>
<tr>
<td>ln(Land)_i</td>
<td>ln(Land)_j</td>
<td>ln(Land)_i</td>
<td>ln(Land)_j</td>
<td>ln(Land)_i</td>
<td>ln(Land)_j</td>
</tr>
<tr>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
</tr>
<tr>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
<td>ln(X)_i</td>
<td>ln(X)_j</td>
</tr>
</tbody>
</table>

Observations | 637,067 | 637,067 | 637,067 | 637,067 | 734128 |
R-squared | 0.7115 | 0.7221 | 0.8125 | 0.4177 | 0.9225 |

Notes: Robust, clustered standard errors are reported in parentheses. *** denotes p<0.01, ** denotes p<0.05, * denotes p<0.1. Trade is the bilateral exports from country i to country j, GDP, and GDP are Gross Domestic Product of country i and j respectively. Dist_i is the distance between country i and j. Eff_i and Eff_j are the environmental efficiency index of country i and j respectively. Pop_i and Pop_j are the population of country i and j respectively. Land_i and Land_j are the land area of country i and j respectively. Contig_i is dummy with that takes 1 if both exporter i and importer j are contiguous and zero otherwise. ComCol_i, j is the dummy with value 1 if both exporter i and importer j share a common language and zero otherwise. CmCol_i is the dummy with value 1 if both have had a common colonizer after 1945 and zero otherwise. All variables that start with “ln” denote the logarithmic transformation of the variable.
### Table 3: Zero Trade Problem

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td></td>
<td>PPML</td>
<td>PPML$_{ij}$</td>
<td>PPML$_{ij,\text{time}}$</td>
<td>PPML$_{ij,\text{time}}$</td>
<td>NB$_{ij,\text{time}}$</td>
<td>HMR$_{ij}$</td>
</tr>
<tr>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
<td>$X_{ij}$</td>
</tr>
<tr>
<td>$\ln GDP_i$</td>
<td>0.6817***</td>
<td>0.6556***</td>
<td>1.2885***</td>
<td>1.2910***</td>
<td>0.6252***</td>
<td>2.3042***</td>
</tr>
<tr>
<td></td>
<td>(0.0033)</td>
<td>(0.0032)</td>
<td>(0.0237)</td>
<td>(0.0001)</td>
<td>(0.0017)</td>
<td>(0.0155)</td>
</tr>
<tr>
<td>$\ln GDP_j$</td>
<td>0.7729***</td>
<td>0.7490***</td>
<td>1.2575***</td>
<td>1.2694***</td>
<td>0.4583***</td>
<td>2.1977***</td>
</tr>
<tr>
<td></td>
<td>(0.0043)</td>
<td>(0.0039)</td>
<td>(0.0219)</td>
<td>(0.0001)</td>
<td>(0.0017)</td>
<td>(0.0158)</td>
</tr>
<tr>
<td>$\ln Dist_{ij}$</td>
<td>-0.5401***</td>
<td>-0.5517***</td>
<td>-0.7803***</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>-1.5927***</td>
</tr>
<tr>
<td></td>
<td>(0.0037)</td>
<td>(0.0035)</td>
<td>(0.0031)</td>
<td></td>
<td></td>
<td>(0.0032)</td>
</tr>
<tr>
<td>$\ln (PER_i/PER_j)$</td>
<td>-0.1796***</td>
<td>-0.2020***</td>
<td>-0.1505***</td>
<td>-0.1677***</td>
<td>-0.3789***</td>
<td>-0.5354***</td>
</tr>
<tr>
<td></td>
<td>(0.0641)</td>
<td>(0.0706)</td>
<td>(0.0553)</td>
<td>(0.0003)</td>
<td>(0.0139)</td>
<td>(0.0386)</td>
</tr>
<tr>
<td>$\ln (TEMP_i/TEMP_j)$</td>
<td>0.0275</td>
<td>0.0211</td>
<td>0.0146</td>
<td>0.0173***</td>
<td>0.0042</td>
<td>-0.0706*</td>
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<tr>
<td></td>
<td>(0.0393)</td>
<td>(0.0386)</td>
<td>(0.0408)</td>
<td>(0.0003)</td>
<td>(0.0145)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\ln Pop_i$</td>
<td>0.1870***</td>
<td>0.1844***</td>
<td>-0.1762***</td>
<td>-0.2177***</td>
<td>-0.2775***</td>
<td>-0.9468***</td>
</tr>
<tr>
<td></td>
<td>(0.0050)</td>
<td>(0.0047)</td>
<td>(0.0581)</td>
<td>(0.0002)</td>
<td>(0.0015)</td>
<td>(0.0308)</td>
</tr>
<tr>
<td>$\ln Pop_j$</td>
<td>0.0915***</td>
<td>0.0881***</td>
<td>-0.3391***</td>
<td>-0.3730***</td>
<td>-0.1335***</td>
<td>-0.5852***</td>
</tr>
<tr>
<td></td>
<td>(0.0040)</td>
<td>(0.0038)</td>
<td>(0.0406)</td>
<td>(0.0002)</td>
<td>(0.0014)</td>
<td>(0.0217)</td>
</tr>
<tr>
<td>$\ln Land_i$</td>
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<td>-0.0708***</td>
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<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
</tr>
<tr>
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<td>(0.0029)</td>
<td>(0.0026)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\ln Land_j$</td>
<td>-0.0460***</td>
<td>-0.0351***</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
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<tr>
<td></td>
<td>(0.0025)</td>
<td>(0.0024)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln Contig_{ij}$</td>
<td>0.7364***</td>
<td>0.7409***</td>
<td>0.4911***</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>-0.0726***</td>
</tr>
<tr>
<td></td>
<td>(0.0133)</td>
<td>(0.0122)</td>
<td>(0.0087)</td>
<td></td>
<td></td>
<td>(0.0103)</td>
</tr>
<tr>
<td>$\ln ComLang_{ij}$</td>
<td>0.2481***</td>
<td>0.2340***</td>
<td>0.1086***</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
</tr>
<tr>
<td></td>
<td>(0.0095)</td>
<td>(0.0087)</td>
<td>(0.0076)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln ComCol_{ij}$</td>
<td>1.6466***</td>
<td>1.5349***</td>
<td>0.1740***</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>0.7774***</td>
</tr>
<tr>
<td></td>
<td>(0.0208)</td>
<td>(0.0190)</td>
<td>(0.0315)</td>
<td></td>
<td></td>
<td>(0.0100)</td>
</tr>
<tr>
<td>$\ln ZHAT$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-3.6547***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0202)</td>
</tr>
<tr>
<td>$\ln INVMILLS$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-2.6261***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.3632)</td>
</tr>
<tr>
<td>Constant</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-20.9285***</td>
<td>-55.3027***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0482)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Observations</td>
<td>665,260</td>
<td>665,260</td>
<td>665,260</td>
<td>661,495</td>
<td>661,495</td>
<td>637,067</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.7051</td>
<td>0.7571</td>
<td>0.8623</td>
<td>-</td>
<td>-</td>
<td>0.8089</td>
</tr>
</tbody>
</table>

**Notes:** Robust standard errors are reported in parentheses. *** denotes p<0.01, ** denotes p<0.05, * denotes p<0.1. Trade is the bilateral exports from country $i$ to country $j$. GDP$_i$ and GDP$_j$ are Gross Domestic Product of country $i$ and $j$ respectively. $Dist_{ij}$ is the distance between country $i$ and $j$. $Eff_i$ and $Eff_j$ are the environmental efficiency index of country $i$ and $j$ respectively. $Pop_i$ and $Pop_j$ are the population of country $i$ and $j$ respectively. $Land_i$ and $Land_j$ are the land area of country $i$ and $j$ respectively. $Contig_{ij}$ is dummy with that takes 1 if both have had a common colonizer after 1945 and zero otherwise. $ComCol_{ij}$ is dummy with value 1 if both exporter $i$ and importer $j$ share a common language and zero otherwise. All variables that start with “$l$” denote the logarithmic transformation of the variable.
### Table 4: Endogenity Problem

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>IVOLS&lt;sub&gt;ij&lt;/sub&gt;</th>
<th>IVGMM&lt;sub&gt;ij&lt;/sub&gt;</th>
<th>SGMM&lt;sub&gt;ij&lt;/sub&gt;</th>
<th>IVPoisson&lt;sub&gt;ij&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnGDP&lt;sub&gt;i&lt;/sub&gt;</td>
<td>1.8041*** (0.0666)</td>
<td>1.8019*** (0.0665)</td>
<td>0.7857*** (0.2170)</td>
<td>1.9713*** (0.0735)</td>
</tr>
<tr>
<td>lnGDP&lt;sub&gt;j&lt;/sub&gt;</td>
<td>1.6621*** (0.0728)</td>
<td>1.6639*** (0.0728)</td>
<td>2.0210*** (0.3501)</td>
<td>1.4566*** (0.1052)</td>
</tr>
<tr>
<td>lnDist&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>-0.6328*** (0.1175)</td>
<td>-1.3397*** (0.0041)</td>
</tr>
<tr>
<td>ln(PER&lt;sub&gt;i&lt;/sub&gt;/PER&lt;sub&gt;j&lt;/sub&gt;)</td>
<td>-0.4559** (0.2083)</td>
<td>-0.4564** (0.1869)</td>
<td>-0.3957*** (0.1386)</td>
<td>-0.5598*** (0.1573)</td>
</tr>
<tr>
<td>ln(TEMP&lt;sub&gt;i&lt;/sub&gt;/TEMP&lt;sub&gt;j&lt;/sub&gt;)</td>
<td>0.7735 (55.2736)</td>
<td>-0.7239 (39.1712)</td>
<td>-0.5294 (2.0297)</td>
<td>-1.5610 (18.9121)</td>
</tr>
<tr>
<td>lnPop&lt;sub&gt;i&lt;/sub&gt;</td>
<td>-0.7716*** (0.1459)</td>
<td>-0.7623*** (0.1455)</td>
<td>-0.3532*** (0.1025)</td>
<td>-0.6197*** (0.0562)</td>
</tr>
<tr>
<td>lnPop&lt;sub&gt;j&lt;/sub&gt;</td>
<td>-0.4386*** (0.1178)</td>
<td>-0.4370*** (0.1170)</td>
<td>-0.5220*** (0.1115)</td>
<td>-0.3333*** (0.0410)</td>
</tr>
<tr>
<td>lnLand&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
</tr>
<tr>
<td>lnLand&lt;sub&gt;j&lt;/sub&gt;</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>(omitted)</td>
</tr>
<tr>
<td>Contig&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>0.0181 (0.0574)</td>
<td>0.4303*** (0.0112)</td>
</tr>
<tr>
<td>ComLang&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>(omitted)</td>
<td>(omitted)</td>
<td>0.1783*** (0.0492)</td>
<td>0.4566*** (0.0180)</td>
</tr>
<tr>
<td>ComCol&lt;sub&gt;ij&lt;/sub&gt;</td>
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<td>(omitted)</td>
<td>0.4268*** (0.1370)</td>
<td>1.3020*** (0.0233)</td>
</tr>
<tr>
<td>m1 p-value</td>
<td>-</td>
<td>-</td>
<td>-3.65</td>
<td>-</td>
</tr>
<tr>
<td>m2 p-value</td>
<td>-</td>
<td>-</td>
<td>1.65</td>
<td>-</td>
</tr>
<tr>
<td>Hansen p-value</td>
<td>2.092 (0.3513)</td>
<td>2.092 (0.3513)</td>
<td>7.3</td>
<td>-</td>
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<td>607,845</td>
<td>614,975</td>
<td>600,957</td>
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<tr>
<td>R-squared</td>
<td>0.3842</td>
<td>0.3863</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors are reported in parentheses. *** denotes p<0.01, ** denotes p<0.05, * denotes p<0.1. Trade is the bilateral exports from country i to country j. GDP<sub>i</sub> and GDP<sub>j</sub> are Gross Domestic Product of country i and j respectively. Dist<sub>ij</sub> is the distance between country i and j. Eff<sub>i</sub> and Eff<sub>j</sub> are the environmental efficiency index of country i and j respectively. Pop<sub>i</sub> and Pop<sub>j</sub> are the population of country i and j respectively. Land<sub>i</sub> and Land<sub>j</sub> are the land area of country i and j respectively. Contig<sub>ij</sub> is dummy with that takes 1 if both exporter i and importer j are contiguous and zero otherwise. ComLang<sub>ij</sub> dummy with value 1 if both exporter i and importer j share a common language and zero otherwise. Comcol<sub>ij</sub> is the dummy with value 1 if both have had a common colonizer after 1945 and zero otherwise. All variables that start with “l” denote the logarithmic transformation of the variable.
APPENDIX-I DATA DESCRIPTION AND SOURCES

_Bilateral exports from country i to country j (X_{ij}):_ Measures the total exports from country $i$ to country $j$ in current period USD. The variable is converted into real terms by export price indices. _Source:_ Direction of Trade Statistics, IMF.

_Gross domestic product of country i and j (GDP_i and GDP_j):_ GDP at purchaser’s prices in million USD. Data are in constant 2005 USD. _Source:_ World Development Indicators, World Bank.

_Distance between country i and j (Dist_{ij}):_ The simple distances calculated following the great circle formula, which uses the latitude and longitude of a country’s most important city (in terms of population) or of its official capital in nautical miles. _Source:_ CEPII Mayer and Zignago (2011) dataset.

_Population of country i and j (Pop_i and Pop_j):_ Total population is based on the _de facto_ definition of population, which counts all residents regardless of legal status or citizenship – except for refugees not permanently settled in the country of asylum, who are generally considered part of the population of their country of origin. The values shown are midyear estimates. _Source:_ World Development Indicators, World Bank.

_Land area of country i and j (Land_i and Land_j):_ Land area is a country’s total area, excluding area under inland water bodies, national claims to a continental shelf and exclusive economic zones. In most cases, the definition of inland water bodies includes major rivers and lakes. _Source:_ World Development Indicators, World Bank.

_Common Language (ComLang_{ij}):_ A dummy variable indicating whether the two countries share a common official language. _Source:_ CEPII Mayer and Zignago (2011) dataset.

_Common Colonizer (ComCol_{ij}):_ A dummy variable indicating whether the two countries had a common colonizer after 1945. _Source:_ CEPII Mayer and Zignago (2011) dataset.

_Contiguity (Contig_{ij}):_ dummy variable indicating whether the two countries are contiguous. _Source:_ CEPII Mayer and Zignago (2011) dataset.
APPENDIX-II EXCHANGE RATE DECOMPOSITION

There are different periodic behaviors of the economic time series and Christiano-Fitzgerald Band Pass Filter (2003) methodology designs a filter that uses the frequency domain to eliminate very slow-moving ("trend") components and very high-frequency (cycle) components of the time series. The principles in the C-F Filter were built on Baxter and King (BK) (1999) filter but their design to approximate the ideal infinite band pass filter is different. Ideal band pass filter constitutes the basis for the C-F Filter. According to "Spectral Representation Theorem" any time series within a broad class can be decomposed into different frequency components. The tool for extracting these components is to use "Ideal Band Pass Filter" which is a linear transformation of the data that leaves intact the components of the data within a specified band of frequencies and eliminates all other components. Consider the decomposition of $x_t$, i.e.;

$$x_t = y_t + \bar{x}_t$$

It is well known;  

$$y_t = B(L)x_t$$

where

$$B(L) = \sum_{j=-\infty}^{\infty} B_j L_j$$

where $L_1 x_t = x_{t-1}$

$$B_j = \frac{\sin(jp) - \sin(ja)}{\pi j}, j \geq 1$$

$$B_0 = \frac{b - a}{\pi}, a = \frac{2\pi}{p_u}, b = \frac{2\pi}{p_l}$$

and we want to isolate the component of $x_t$ with a period of oscillation between $p_l$ and $p_u$, where $2 \leq p \leq p_u < \infty$. However, the application of ideal band pass filter requires infinite data.

For finite set, C-F filter solves the following minimization problem and estimate $\hat{B}^{p,f}_{j,15}$:

$$\min E[(y_t - \hat{y}_t)^2 | x] s.t \hat{y}_t = \sum_{j=-p}^{f} \hat{B}^{p,f}_{j} x_{t-j}$$

12 We define the trend as the permanent component of exchange rate and cycle as the temporary component of exchange rate.
13 See Cramer and Leadbetter (1967) and Lippi(2001) for a formal analysis.
15 See Christiano and Fitzgerald (2003) for the solution and further details.
This problem can be expressed in the frequency domain by exploiting the standard frequency domain representation for a variance.\textsuperscript{16}

\[
\min \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \hat{B}^{p,f}(e^{-iw}) - B \left( (e^{-iw}) \right) \right|^2 f_x(w) dw,
\]

Here, \( f_x(w) \) is the spectral density of \( x_t \), and

\[
\hat{B}^{p,f}(L) = \sum_{j=-p}^{f} \hat{B}_j^{p,f} L^j
\]

C-F filter differs from B-K Filter in three aspects. First, in C-F Filter the presence of \( f \) indicates that the solution to the minimization problem depends on the properties of the time series representation of \( x_t \). Second, \( \hat{B}_j^{p,f} = 0 \) is never imposed as a constraint. Third, C-F Filter uses all the data for each \( t \), and \( p \) and \( f \) vary with \( t \) and different from each other.

\textsuperscript{16} See Sims (1972) for details.